Learning about Expected Profitability

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Abstract

Asset-pricing theories price expected profitability, yet empirical work typically proxies it with current profitability. We build a theory-consistent measure of expected profitability revisions using a structural learning model in which investors update beliefs about a latent profitability component from firm earnings and an industry profitability signal. The filter yields a firm-level expected change in profitability and decomposes it into short-run updating and long-run anchoring components. Both components predict returns, and decile portfolios sorted on expected profitability revisions earn economically large, monotone spreads that are strongest among small and young firms. The return premium is not subsumed by current profitability or momentum and is about 3.2% per year. Consistent with the learning mechanism, uncertainty-scaled earnings innovations are significantly priced in the cross section. Bayesian factor inclusion tests (Chib et al., 2020) select a compact four-factor set that includes our expected profitability revision factor.

Keywords: Belief updating, profitability expectations, return predictability, information processing, earnings surprises, cross-section of returns

JEL codes: G12, G14, D83, C58, E44

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1. Introduction

Understanding firm profitability is a central theme in financial economics, with far-reaching implications for asset pricing, corporate finance, and the efficient allocation of capital in the economy. Profitability is a key metric for evaluating a firm's financial health, growth prospects, and ability to generate shareholder value. Importantly, in both the Fama-French and q-factor models, the object that prices equity risk is expected profitability. Yet, empirical implementations typically proxy expected with current profitability. This disconnect motivates our study.

Our approach is measurement-driven. We build a structural learning framework in which firm profitability is driven by a latent component and investors update beliefs using two accounting-based signals: realized firm profitability and an industry profitability signal. The resulting filter delivers a firm-level estimate of expected future profitability and a decomposition of expected profitability revisions into a short-run belief component and a long-run anchor component. We estimate the learning parameters using only observables at each period and exclude price-based signals so that the expectation proxy is not mechanically contaminated by returns.

Consistent with the model's learning mechanism, we find direct cross-sectional evidence that investors update beliefs and that these belief updates are priced. The key prediction of the learning model is that the price impact of a given surprise should be larger when posterior uncertainty is high, because the Kalman gain scales belief revisions. Empirically, we find that the interaction terms between posterior uncertainty and the firm-level surprise shocks carry positive and statistically significant premia across specifications. These results are robust to controlling for profitability levels and a rich set of firm characteristics, and they support the interpretation that expected profitability revisions embedded in our filter reflect economically meaningful learning about a latent profitability driver rather than mechanical transformations of observed accounting variables.

This paper develops a real-time, theory-consistent measure of expected profitability and expected profitability revisions implied by a structural learning model. The key empirical object is the model-implied expected change in profitability relative to its current level, which captures belief revisions about a latent profitability driver inferred from observable earnings-based profitability and industry signals. We show that both the short-run updating component and the long-run anchoring component of expected profitability predict stock returns, with economically large and monotone return spreads that are strongest among small and young firms. The predictability is not subsumed by current profitability or standard momentum controls, and it remains informative relative to leading multifactor benchmarks. Finally, Bayesian factor inclusion tests identify a compact set of non-market factors that includes our expected profitability revision factor, suggesting that belief revisions about fundamentals contain distinct pricing information.

The paper makes three contributions. First, it provides a firm-level proxy for profitability expectations and their revisions implied by an explicit learning mechanism and estimated from accounting information. Second, it documents an expected profitability revision premium, showing that revisions in expected profitability forecast returns beyond current profitability and leading factor models, with the strongest effects concentrated where information frictions are more severe. Third, it connects the return predictability to factor pricing by conducting spanning tests and Bayesian model comparison, which quantify how expected profitability revisions relate to, and complement, existing factor structures.

Empirically, we show that the expected change in profitability forecasts returns beyond current profitability and beyond standard factor models. In one-way decile sorts on the expected change in profitability, average excess returns rise monotonically from 0.57% per month in the lowest decile to 0.92% per month in the highest decile, producing a high-minus-low spread of 0.35% per month with a Newey-West t-statistic of 2.84. The spread is substantially larger among micro-cap firms (0.61% per month, t = 3.57) and remains sizable for small firms (0.31% per month, t = 2.70), as well as large firms (0.32% per month, t = 2.44), consistent with stronger learning frictions in micro-cap and smaller firms.

In factor tests, a traded factor built from expected profitability revisions earns a sizable and statistically reliable premium, and this premium remains largely intact after controlling for standard benchmark factor models. Across FF6 and q-factor variants, the factor's abnormal return is consistently positive and economically meaningful, with alphas in a narrow range of roughly 0.22 to 0.27% per month. The same pattern holds when we decompose expected profitability revisions into short-run and long-run components: both contribute positively, but the long-run component is consistently the stronger and more robust source of explanatory power. Cross-sectional regressions deliver a complementary message. Firms with larger expected profitability revisions earn higher subsequent returns even after controlling for standard predictors, and this relationship is driven primarily by revisions in longer-horizon expectations rather than by transitory short-run updates. Finally, the predictive content of expected profitability revisions persists over multiple months, whereas the predictive power of current profitability fades more quickly, consistent with a gradual repricing of longer-run fundamental news.

In addition, we use Bayesian factor inclusion tests based on the marginal-likelihood model scan of Chib et al. (2020) to quantify which subset of candidate factors is most consistently supported as priced in the data. Conditional on always including the market factor, the maximum-posterior specification selects a compact four-factor set among the non-market candidates, namely HML, R_IA, R_EG, and ΔExpProf. The full-sample posterior inclusion probabilities reinforce this concentration: R_EG and ΔExpProf have inclusion probability 1.00 and R_IA about 0.89, while the remaining factors receive materially lower support.

The recursive version of these tests yields the same qualitative message over time: once sufficient data accumulate, R_EG and ΔExpProf rise to and remain essentially at inclusion probability one, whereas the other factors display weaker and more episodic posterior support. These inclusion results complement the spanning evidence by showing that the expected profitability change factor is not only statistically strong in standard regressions, but is also consistently selected by model comparison procedures that aggregate evidence across the full model space.

Our findings have important implications for investors, managers, and policymakers alike. For investors, recognizing how markets learn from earnings can sharpen trading strategies and reveal mispricing opportunities created by informational frictions. For corporate managers, the results emphasize the value of transparent communication: firms that consistently meet expectations and clearly convey profitability prospects are rewarded with higher valuations and lower costs of capital. For policymakers, the evidence highlights the central role of timely and accurate disclosure in fostering market efficiency and stability, reducing information asymmetries, and supporting the effective allocation of capital across the economy.

Section 2 reviews related literature. The remainder of the paper develops the model and estimation in Section 3 and Section 4, respectively, presents our empirical results in Section 5, and concludes in Section 6.

2. Related Literature

This paper contributes to several connected literatures at the intersection of learning about fundamentals, expected cash flows, and cross-sectional asset pricing. Our central object is a real-time measure of investors' expected profitability and, in particular, revisions in expected profitability generated by a structurally disciplined learning framework. Our study also relates to the large literature on profitability-based return predictors, the information content of earnings, and models with information frictions and gradual diffusion of industry information. By grounding the analysis in a structural filter estimated on accounting data, we bring these strands together and clarify what, exactly, markets appear to be learning about firm fundamentals and how that learning affects expected returns.

Our approach is most closely related to the learning-based valuation framework of Pástor and Veronesi (2003), who show that uncertainty about profitability and the process of learning about fundamentals can have first-order effects on prices and expected returns. While Pástor and Veronesi (2003) emphasize how learning and parameter uncertainty shape valuation dynamics, our contribution is empirical: we construct firm-level beliefs about a latent profitability driver using an observable accounting signal and rolling-window structural estimation, and we quantify how updates to those beliefs map into cross-sectional return predictability. In this sense, the paper provides a measurement counterpart to the learning mechanism by delivering an implementable proxy for expected profitability that is naturally implied by the model's updating equations, and by isolating the belief-revision component that is central to learning-based asset-pricing predictions.

A closely related mechanism appears in Dew-Becker, Giglio, and Molavi (2025), who argue that many higher-order features of returns and volatility emerge endogenously when prices reflect Bayesian (or biased) filtering of signals about a latent value. In their framework, time variation and asymmetry arise because the "gain" on news scales with posterior uncertainty and is shaped by non-Gaussian beliefs. We adopt an analogous filter-based structure in the cross section: investors learn about latent expected profitability μ_{it} from firm and industry signals, so revisions in expected profitability inherit state dependence through ν_{it} and become predictably priced.

A growing body of research has explored the information content of firm earnings and their impact on stock prices, emphasizing earnings as a key signal of future profitability (Kothari, 2001; Bernard and Thomas, 1989; Brown and Niederhoffer, 1968). At the same time, the degree to which investors learn from earnings and update beliefs about a firm's future profitability remains an open question. Traditional asset pricing models often assume perfect information and rational expectations, but in practice investors face information frictions and may exhibit bounded rationality when processing financial information (Daniel, Hirshleifer, and Subrahmanyam, 1998; Barberis, Shleifer, and Vishny, 1998). Our analysis speaks directly to these themes by using a structural filter to map observed earnings-based profitability into a time series of beliefs and belief revisions, and by studying how the pricing of those revisions varies across firms with different information environments and signal precision.

A related and expanding literature highlights that investors' expectations and expectation errors contain information about discount rates and cash-flow beliefs that is not captured by realized fundamentals alone. Greenwood and Shleifer (2014) study expectations of returns and the mapping between survey-based expectations and realized returns, emphasizing that expectations data can be informative about variation in required returns. Delao and Myers (2021) and Delao and Myers (2024) formalize and quantify the distinct roles of subjective cash-flow and discount-rate expectations for explaining asset prices. Our empirical strategy complements these approaches by producing a firm-level expectation measure from standard financial statements through a disciplined learning structure. Relative to survey-based or directly elicited expectations, our measure has broad firm coverage and a transparent decomposition into a short-run updating component and a long-run anchor component, allowing us to test which part of expectations is most strongly associated with return predictability. At the same time, our results reinforce the broader message of Delao and Myers (2021, 2024): separating cash-flow expectations from discount-rate variation is empirically central, and profitability-revision measures provide a natural laboratory for that separation.

Our empirical tests are benchmarked against leading multifactor models that incorporate profitability- and investment-based factors. Fama and French (2015) introduce a five-factor model in which profitability and investment play key roles alongside the market, size, and value factors, while Hou et al. (2015) propose an investment-based framework that digests a broad set of anomalies and provides a prominent alternative factor structure. These models discipline the interpretation of new predictors by asking whether return spreads reflect compensation for exposures already captured by standard factors. We therefore evaluate whether expected profitability revisions generate abnormal returns relative to these benchmarks and whether adding our belief-revision factor improves pricing performance. Conceptually, our contribution is complementary to profitability and investment factors built from realized accounting characteristics: we focus on expected profitability and its revisions, which are natural state variables in learning models and need not be spanned by static characteristic sorts.

A central challenge in empirical asset pricing is to distinguish whether a new predictor represents priced risk or mispricing, and to assess whether it contains incremental information beyond existing characteristics. Green et al. (2017) systematically study which characteristics provide independent information about average stock returns, underscoring the importance of careful incremental tests and the risk of redundant predictors. Kelly et al. (2019) provide a unifying framework in which characteristics proxy for covariances with priced shocks, offering a risk-based interpretation for characteristic predictability. Our analysis speaks directly to these themes. First, by constructing an expected-profitability revision measure from an explicit learning structure, we generate a predictor with a precise economic interpretation: it is the innovation to beliefs about the latent profitability driver implied by the model. Second, we evaluate incremental explanatory power relative to standard characteristic controls and factor benchmarks, consistent with the identification concerns emphasized in Green et al. (2017). Third, we assess whether the return premium associated with expected profitability revisions is consistent with systematic exposure to shocks, as in Kelly et al. (2019), or instead reflects delayed incorporation of information by examining how predictability varies with proxies for information frictions and signal precision.

Overall, the paper bridges learning-based valuation theory (Pástor and Veronesi, 2003) and earnings-based evidence on information content (Kothari, 2001; Bernard and Thomas, 1989; Brown and Niederhoffer, 1968) with empirical work on expectations (Greenwood and Shleifer, 2014; Delao and Myers, 2021, 2024), modern multifactor benchmarking (Fama and French, 2015; Hou et al., 2015), and the literature on characteristics and interpretation (Green et al., 2017; Kelly et al., 2019). Our main contribution is a real-time, structurally motivated measure of expected profitability and its revisions, and evidence that these belief updates are economically and statistically meaningful for cross-sectional stock return predictability in settings with information frictions.

3. Expected firm profitability

Profitability is a central state variable in asset pricing models. Building on this tradition, we introduce a structural learning framework in which expected future profitability is shaped not only by observed earnings but also by investor beliefs about unobservable drivers of firm fundamentals.

Let firm i's operating profits at time t be

$$\Pi_{it} = X_{it} A_{it}, \tag{1}$$

where A_{it} is the stock of productive assets and X_{it} denotes cash flows per unit of asset, our measure of profitability. As in Liu, Whited, and Zhang (2009) and Hou, Mo, Xue, and Zhang (2021), the firm chooses investment I_{it} to maximize the market value of equity

$$V_{it} = \max_{I,A} \mathbb{E}_t \sum_{s=0}^{\infty} M_{t+s} D_{i,t+s}, \tag{2}$$

where M_t is the stochastic discount factor and D_{it} are firm cash flows accounting for operating profits less adjustment costs and investment expenditures: $D_{it} = X_{it} A_{it} - \frac{a}{2} \left(\frac{I_{it+1}}{A_{it+1}} \right)^2 A_{it} - I_{it}$.

The first-order condition implies that expected investment returns depend on expected future profitability, expected marginal q, $\mathbb{E}_t(q_{it+1})$, and expected adjustment cost, $\mathbb{E}_t\left(\left(\frac{I_{it+1}}{A_{it+1}}\right)^2\right)$.

$$\mathbb{E}_t(R^I_{it+1}) = \frac{\mathbb{E}_t(X_{it+1}) + (1-\delta)\mathbb{E}_t(q_{it+1}) + \frac{a}{2}\mathbb{E}_t\left(\left(\frac{I_{it+1}}{A_{it+1}}\right)^2\right)}{q_{it}} \tag{3}$$

Our empirical focus is one-period-ahead expected profitability, $\mathbb{E}_t[X_{i,t+1}]$. Because q is the discounted present value of profitability beyond t+1, the same forces that move $\mathbb{E}_t[X_{i,t+1}]$ also likely shift $\mathbb{E}_t[q_{i,t+1}]$. Targeting $\mathbb{E}_t[X_{i,t+1}]$ therefore provides a conservative lower bound on learning about fundamentals and the resulting return effects.

For parsimony, we treat the expected adjustment-cost term $\frac{a}{2}\,\mathbb{E}_t[(I_{i,t+1}/A_{i,t+1})^2]$ as second-order and absorb it with standard investment controls; results are unchanged when we add explicit proxies.

Expected marginal q and investment growth co-move with expected profitability but with attenuated sensitivity: q averages expectations across horizons, and investment adjusts with costs, constraints, and strategic frictions. Empirically, profitability, q, and investment are positively related, with effects weakening from profitability to investment (Liu et al., 2009; Hou et al., 2021). Thus, one-period ahead expected profitability remains the primary driver of expected returns in our setting.

3.1 Learning about firm profitability drivers

We model firm profitability as a latent mean-reverting process observed imperfectly through firm earnings and an industry-wide signal. Investors filter these signals to form beliefs about underlying profitability dynamics, and these beliefs govern expected profitability revisions.

Let X_{it} denote firm i's profitability at time t, driven by an unobservable component μ_{it}

that evolves as an Ornstein–Uhlenbeck process:

$$dX_{it} = \lambda_i(\mu_{it} - X_{it})dt + \sigma_{iX} dW_{it}^X, \tag{4}$$

where λ_i is the speed of mean reversion, determining how quickly X_{it} adjusts towards the latent profitability driver μ_{it}, and σ_{Xi} is the volatility of realized profitability. Importantly, while X_{it} is observable, the fundamental driver of profitability, μ_{it}, is unobservable and follows its own stochastic, mean-reverting process as in Andrei, Mann, and Moyen (2019).

$$d\mu_{it} = \kappa_i(\bar{\mu}_i - \mu_{it})dt + \sigma_{\mu i} dW_{it}^{\mu}, \tag{5}$$

where $\kappa_i > 0$ is the speed of mean reversion, $\bar{\mu}_i$ is the long-run anchor, σ_{μ_i} is the volatility of the latent driver, and W_{it}^{μ} is a standard Brownian motion.

Agents learn about the latent profitability driver μ_{it} from observed profitability X_{it} and a noisy industry signal s_{it}:

$$s_{it} = \mu_{it} + \epsilon_{it}^s, \tag{6}$$

where $\epsilon_{it}^s \sim N(0, \sigma_{is}^2)$. Using all available information at time t, summarized in the information set $\mathcal{F}_t$, agents form their posterior beliefs about μ_{it}, denoted as $\hat{\mu}_{it} = \mathbb{E}(\mu_{it}|\mathcal{F}_t)$.

Proposition 3.1. *(Filtering of profitability beliefs). Given observed firm profitability X_{it} and industry signal s_{it}, investors' posterior belief about the unobservable profitability driver μ_{it} evolves as*

$$d\hat{\mu}_{it} = \kappa_i(\bar{\mu}_i - \hat{\mu}_{it})dt + \frac{\lambda_i \nu_{it}}{\sigma_i^X} d\widetilde{W}_{it}^X + \frac{\nu_{it}}{\sigma_i^s} d\widetilde{W}_{it}^s, \tag{7}$$

with posterior variance dynamics

$$d\nu_{it} = \left(\sigma_{\mu_i}^2 - 2\kappa_i \nu_{it} - \frac{\lambda_i^2}{(\sigma_i^X)^2}\nu_{it}^2 - \frac{1}{(\sigma_i^s)^2}\nu_{it}^2\right) dt. \tag{8}$$

The Filtering Theorem applied to derive the belief formation dynamics is discussed in Appendix B.

Agents update their beliefs about μ_{it} based on surprises in realized profitability and the industry signal. These surprises, denoted $d\tilde{W}_t^{Xi}$ and $d\tilde{W}_t^{is}$, are standard Brownian shocks:

$$d\tilde{W}_{it}^{X} = \frac{dX_{it} - \lambda_i(\hat{\mu}_{it} - X_{it})dt}{\sigma_{iX}}, \tag{9}$$

and

$$d\tilde{W}_{it}^{s} = \frac{ds_{it} - \hat{\mu}_{it}dt}{\sigma_{is}}. \tag{10}$$

Proposition 3.2. *(Expected profitability decomposition). For any $\tau > t$, the conditional expectation of profitability is a weighted average of current profitability, the filtered short-run component, and the long-run anchor:*

$$E_t[X_{i,\tau}] = a_i\, X_{it} + b_i\, \hat{\mu}_{it} + c_i\, \bar{\mu}_i, \tag{11}$$

where

$$a_i = e^{-\lambda_i(\tau-t)}, \quad b_i = \frac{\lambda_i}{\lambda_i - \kappa_i}\left(e^{-\kappa_i(\tau-t)} - e^{-\lambda_i(\tau-t)}\right), \quad c_i = 1 - a_i - b_i.$$

To avoid ambiguity, we use the term expected profitability revision to refer to the model-implied expected change in profitability relative to its current level, denoted $\Delta\text{ExpProf}_{it} = E_t[X_{i,\tau}] - X_{i,t}$. We use expected profitability to refer to the level expectation implied by the filter, denoted $\text{ExpProf}_{it} = E_t[X_{i,\tau}]$.

Proposition 3.2 shows how investors form expectations about future profitability $X_{i,t+\tau}$. Current profitability X_{it} serves as the starting point, but its influence decays over time at a rate governed by λ_i. Beyond this fading anchor, two forces shape expected changes in profitability. The first is mean reversion, which pulls profitability toward the long-run level $\bar{\mu}_i$. The second is learning: as investors update their beliefs about the latent driver $\hat{\mu}_{it}$, these revisions feed directly into expectations. Together, the dynamics of X_{it}, the belief component $\hat{\mu}_{it}$, and the long-run anchor $\bar{\mu}_i$ determine how forward-looking expectations evolve.

Much of the empirical asset-pricing literature, within both Fama–French and modern Q implementations, uses current profitability as a proxy for expected profitability. Our framework shows that revisions in expected profitability, i.e., short- and long-run updates generated by the learning process, carry incremental predictive power for returns that the current profitability level alone cannot capture. By construction, measures that rely on current profitability only omit these expectation updates. Factor regressions that rely solely on current profitability, therefore mis-measure the state variable that should enter pricing relations. Replacing or augmenting the level with our revision-based measure aligns measurement with theory, strengthens return predictability, and improves model fit.

The learning parameters have direct economic interpretations that guide the empirical analysis. The parameter λ_i governs how quickly realized profitability X_{it} reverts toward its long-run level: higher λ_i implies faster adjustment following profitability shocks, while lower λ_i implies more persistent deviations. The parameter κ_i governs the persistence of the latent profitability driver μ_{it}: higher κ_i implies that shocks to fundamentals dissipate more quickly, while lower κ_i implies a more persistent driver and therefore greater scope for belief revisions to accumulate over time. The expected profitability revision $\Delta\text{ExpProf}_{it}$ is the model-implied innovation to beliefs about fundamentals reflected in expected profitability relative to the current level; it is the object that the learning mechanism predicts should be informative for prices and expected returns.

3.2 Benchmark without learning

To isolate and quantify the impact of pure learning on asset prices, we compare a setting where firm profitability is fully observable to one where agents update their beliefs based on earnings surprises. Consider the case where the profitability factor μ_{it} is fully observable from the signal, i.e., $\mu_{it} = s_{it}$, which implies $\sigma_{is} = 0$. In this scenario, the expected profitability at time τ, $E_t(X_{i\tau})$, is determined entirely by the current profitability level X_{it}, the current profitability factor μ_{it}, and the expected mean reversion:

Proposition 3.3 (No-learning benchmark)**.** *If profitability drivers are fully observable (μ_{it} known), then*

$$E_t[X_{i,\tau}] = e^{-\lambda_i(\tau-t)}X_{it} + (1 - e^{-\lambda_i(\tau-t)})\mu_{it}. \tag{12}$$

In the no-learning benchmark, earnings surprises have no effect on μ_{it}, and consequently, they do not influence $E_t(X_{i\tau})$, since μ_{it} is directly observable and evolves according to:

$$d\mu_{it} = \kappa_i(\bar{\mu}_i - \mu_{it})dt + \sigma_{\mu i}dW_{it}^{\mu}. \tag{13}$$

In the absence of learning, only current profitability and the observable driver matter for expectations. Comparing this to Proposition 3.1 highlights that with learning, revisions to beliefs $\hat{\mu}_{it}$, i.e., earnings surprises $d\tilde{W}_{it}^{x}$ and industry-level surprises $d\tilde{W}_{it}^{s}$, become a priced component of expected returns.

3.3 Testable predictions

The model yields three central predictions for empirical analysis. First, the model predicts that revisions in expected profitability, rather than current profitability alone, forecast stock returns. Because investors must learn about the latent drivers of profitability, belief updates about $\hat{\mu}_{it}$ shift expected future profitability and therefore expected returns. This implies that both short-run adjustments and long-run revisions to expected profitability are priced in the cross-section. Firms experiencing upward revisions in profitability expectations should earn higher subsequent returns.

Second, the magnitude of this expected profitability revision premium should vary systematically across firms depending on the precision of information available. In the model, posterior uncertainty ν_{it} governs how strongly new signals move beliefs. When uncertainty is high, as is typically the case for small firms or firms with limited information production, profitability news induces larger belief revisions and larger price effects. By contrast, when information is abundant and beliefs are precise, revisions in expected profitability carry less incremental predictive power. The model therefore predicts stronger return premia among firms facing greater information frictions.

Third, the model predicts that expected profitability change is a priced risk factor in aggregate, not merely a firm-level predictor. Aggregating firms by expected profitability revisions should produce a traded factor with a sizable average return and a large high-minus-low spread, reflecting compensation for systematic variation in belief-updating shocks. In other words, the same learning-driven revisions that forecast returns in the cross-section should also appear as a factor premium in the time series.

Corollary 3.4 (State dependence of updating)**.** *Belief revisions are larger when posterior uncertainty ν_{it} is high and when signals are precise (low σ_i^s). The impact of learning on equity returns is stronger for firms with scarce information (e.g., small/young/low-coverage) and firms operating within industries with stable profitability.*

Corollary 3.4 suggests that the weight placed on new information is larger precisely when prior uncertainty is high and the signal is clean. In our setting, the load of signal surprises in $d\hat{\mu}_{it}$ scales with ν_{it}/σ_i^s, so a given earnings or industry-news surprise shifts beliefs more when investors know little (large ν_{it}) and the news is precise (low σ_i^s). Small, young, or thinly covered firms typically have higher posterior uncertainty, so identical shocks generate larger revisions to expected profitability and, through discount-rate adjustments, stronger return effects. Likewise, industries with stable profitability produce higher signal-to-noise news about μ_{it}, amplifying belief updates and their pricing impact. When uncertainty is low or signals are noisy, updates are muted, and the learning channel's contribution to expected returns diminishes.

To empirically distinguish the impact of learning from the effects of mean-reverting drivers of profitability, we test whether the belief updating terms $\nu_{it} d\tilde{W}_{it}^{x}$ and $\nu_{it} d\tilde{W}_{it}^{s}$ are priced in the cross-section of stock returns after controlling for current firm- and industry-level profitability, as well as the mean-reversion parameters λ_i and κ_i. If these shocks are priced, it would indicate that firm profitability μ_{it} is not fully observable from industry-level profitability signals, confirming that learning plays a significant role in shaping investor beliefs about firm fundamentals and, consequently, asset prices.

We test this implication directly in Table VIII. In monthly Fama–MacBeth regressions, the interaction terms between posterior uncertainty and the model-implied surprise innovations, $\nu_{it} d\widetilde{W}_{it}^{x}$, earn positive and statistically significant premia across specifications, consistent with belief updating being priced.

It is important to emphasize that we are not proposing a new asset pricing model or a new set of preference-based pricing kernels. Instead, our framework is firmly grounded in the existing setup where expected profitability is a key state variable for asset prices. In our setting, signed earnings surprises scaled by uncertainty govern how investors update their beliefs about future profitability, and it is these belief updates, operating through the expected profitability channel, that are priced in the cross-section. We do not model agent preferences or introduce new sources of risk aversion. Rather, our contribution is to show that the interaction of earnings news with posterior uncertainty generates systematic, economically meaningful variation in expected profitability that is reflected in returns.

4. Data & Estimation

4.1 Data

We start our sample in January 1972 due to the availability of earnings announcement data and book equity in Compustat quarterly files. Return data comes from CRSP. Control variables are obtained from `openassetpricing.com` and factor data comes from Ken French and Lu Zhang's websites. Our sample ends in 2021.

Following Hou, Xue, and Zhang (2015), we measure profitability as ROE, computed as income before extraordinary items (Compustat quarterly item IBQ) divided by 1-quarter-lagged book equity. As in Davis, Fama, and French (2000), book equity is shareholders' equity, plus balance-sheet deferred taxes and investment tax credit (item TXDITCQ) if available, minus the book value of preferred stock. Depending on availability, we use stockholders' equity (item SEQQ), or common equity (item CEQQ) plus the carrying value of preferred stock (item PSTKQ), or total assets (item ATQ) minus total liabilities (item LTQ). We use redemption value (item PSTKRQ) if available, or carrying value for the book value of preferred stock.

4.2 Industry signal

In our model, firm profitability X_{it} is driven by a latent profitability driver μ_{it}, and investors form beliefs about μ_{it} using an observable signal s_{it}. A suitable signal should be (i) observable in real time and (ii) economically related to the level toward which firm profitability tends to converge. Industry-level profitability is a natural candidate, consistent with a large accounting and asset-pricing literature that uses industry profitability as a long-run anchor for firm earnings.

In the data, we define the signal s_{it} as the median profitability (ROE) among firms in firm i's industry, computed at quarter t using only information contained in $\mathcal{F}_{it}$. We implement industries using the Fama-French 30 classification (and verify robustness to alternative classifications). Importantly, the industry median is computed in real time: at each quarter t, we take the set of industry firms for which quarterly accounting data are observable by quarter t, compute their ROE, and form the cross-sectional median. This construction ensures that s_{it} is an observable accounting-based signal available when the belief update at t is formed.

Existing accounting literature explicitly uses industry profitability as a long-run anchor for firm-level earnings. Gebhardt et al. (2001) implement a residual income model in which firm-level ROE is assumed to mean revert linearly to the industry median ROE, defined as the moving median ROE over the prior 5-10 years for profitable firms in the same Fama-French industry. Similarly, Gode and Mohanram (2003) impose convergence of firm ROE to an industry-median benchmark when constructing implied cost-of-capital estimates. Empirical asset-pricing work likewise emphasizes the importance of industry structure and industry-wide fundamentals. Hou (2007) find that, within the same industry, big firms lead small firms; firms in more concentrated industries earn lower returns even after controlling for standard return determinants (Hou and Robinson, 2006); and Moskowitz and Grinblatt (1999) document that stock-level momentum profits are much weaker once industry momentum is controlled for. Novy-Marx (2013) show that trends in firm profitability forecast stock returns and that industry earnings trends help shape future firm-level profitability and investment, while Hou et al. (2021) demonstrate that industry earnings contain predictive information about firm-level profitability, reinforcing the role of industry-wide accounting data as a key signal for firm fundamentals.

These studies collectively support the assumption that industry-level profitability serves as a relevant and informative signal in shaping firm-level profitability expectations. By incorporating learning mechanisms into our model, we aim to quantify how industry-wide earnings influence firm-specific profitability beliefs and, in turn, asset prices.

Specifically, we define this observable signal to be the median industry-level ROE, estimated for each firm i at each quarter t. We use the Fama–French 30 industry classification as the primary categorization scheme.[1] This classification groups firms by their primary business activities and market characteristics, ensuring that firms within an industry share common external influences such as supply chain dynamics, regulatory environments, and technological advancements. These shared factors drive industry-level profitability trends, making industry profitability a natural anchor for firm-level profitability drivers.

Furthermore, industry profitability is time-varying, reflecting changing market conditions, competitive pressures, and shifts in industry-specific profitability drivers. For instance, industries sensitive to macroeconomic conditions, such as manufacturing or consumer goods, often exhibit profitability patterns that mirror broader economic cycles. Similarly, technology-driven industries may experience profitability shifts due to innovation or market disruptions. The dynamic nature of industry profitability ensures it remains a relevant benchmark for individual firms' profitability, capturing both systematic factors and industry-specific trends to which firms' profitability levels tend to adjust over time. By using industry profitability as the signal s_{it}, our model effectively incorporates these empirically supported relationships into the belief formation process.

4.3 Firm-level estimation

We estimate firm-specific learning parameters using accounting data only. For each firm i and each estimation date t, we estimate the parameter vector

$$\theta_{it} \equiv (\lambda_{it}, \kappa_{it}, \sigma_{\mu,it}),$$

[1]For robustness, we also test the Fama–French 12 industry classification and Fama–French 12 and size (small versus large firms) classification. Our results remain robust to the change in classification.

by matching the firm- and industry-level cash-flow dynamics implied by the learning model to observed moments computed from data available at t. All estimation is conducted in rolling windows that end at t, so that both inputs and targets are measurable with respect to $\mathcal{F}_{it}$ and no information beyond t enters.

Expanding-window design. Let $\mathcal{W}_{it}$ denote the estimation window for firm i ending at quarter t (an extending-length window). We re-estimate parameters on a rolling basis at a coarse frequency (every two years, i.e., every eight quarters) and carry the latest estimate forward between re-estimation dates. Thus, at any quarter t there exists a most recent estimation date $\tau(t) \leq t$ such that $\theta_{i,\tau(t)}$ is the parameter vector used for belief updates at t.

Target moments and SMM objective. Within each window $\mathcal{W}_{it}$, we compute three empirical moments: (i) the autocorrelation of firm ROE, (ii) the autocorrelation of industry ROE, and (iii) the covariance between firm and industry ROE. For a candidate parameter vector θ, we simulate the model-implied counterparts of these moments and choose θ_{it} to minimize the weighted distance between simulated and empirical moments:

$$\theta_{it} \in \arg\min_{\theta \in \Theta} \left(m_{it}^{\text{sim}}(\theta) - m_{it}^{\text{data}}\right)' W_{it} \left(m_{it}^{\text{sim}}(\theta) - m_{it}^{\text{data}}\right),$$

where m_{it}^{data} is computed using data in $\mathcal{W}_{it}$ only, and $m_{it}^{\text{sim}}(\theta)$ is computed from simulated paths under θ using the same window length.

Initialization using window-level moments (real-time). To initialize the numerical search, we use window-level estimates computed within $\mathcal{W}_{it}$:

- Firm-specific cash-flow volatility $\sigma_{X,it}$ is set to the sample standard deviation of realized ROE $X_{i\tau}$ over $\tau \in \mathcal{W}_{it}$.
- The steady-state profitability level $\bar{\mu}_{it}$ is set to the sample mean of $X_{i\tau}$ over $\tau \in \mathcal{W}_{it}$.

- The volatility of the observed signal $\sigma_{s,it}$ is set to the sample standard deviation of $s_{i\tau}$ over $\tau \in \mathcal{W}_{it}$.

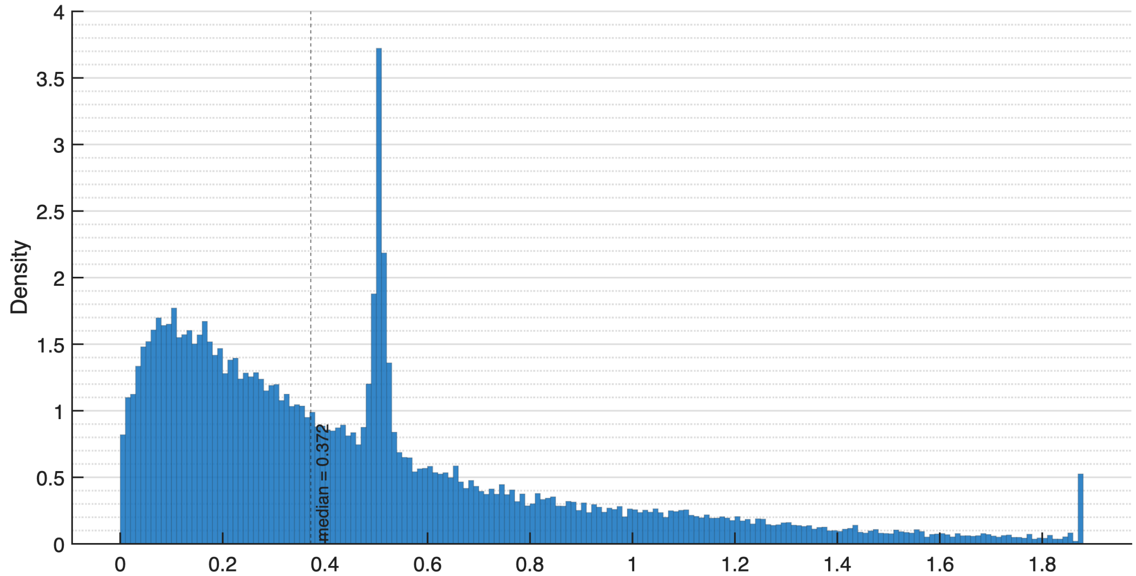
- Initial mean-reversion speeds $(\lambda_{0,it}, \kappa_{0,it})$ are obtained from AR(1) OLS regressions for firm and industry ROE within $\mathcal{W}_{it}$.

These initial values use only data observed by t. We restrict the parameter search to a plausible set Θ by dropping extreme mean-reversion rates $(\lambda_{0,it}, \kappa_{0,it})$ below 0 and above 5 to ensure numerical stability.

Why we do not target the first two moments of ROE. We do not include the mean and variance of firm-level ROE as SMM target moments because they are mechanically pinned down by the window-level calibration of $(\bar{\mu}_{it}, \sigma_{X,it})$. Including them would double count information already embedded in the initialization and would overweight moments that are not informative about $(\lambda_{it}, \kappa_{it}, \sigma_{\mu,it})$ conditional on $(\bar{\mu}_{it}, \sigma_{X,it})$.

Figure 1 summarizes the cross-sectional distributions of the estimated mean-reversion speeds, λ_i and κ_i, obtained from our rolling-window structural estimation. The histogram for λ_i captures heterogeneity in how rapidly realized firm profitability X_{it} reverts toward its long-run level: firms with larger λ_i exhibit faster adjustment of ROE following profitability shocks, whereas firms with smaller λ_i display more persistent deviations from their long-run profitability. The histogram for κ_i reflects heterogeneity in the persistence of the latent profitability driver μ_{it} that agents seek to learn about. Larger values of κ_i imply that the latent driver mean-reverts more quickly (so shocks to fundamentals dissipate faster), while smaller values imply a more persistent driver, increasing the scope for belief revisions to accumulate over time. Overall, the dispersion in both histograms indicates substantial cross-sectional variation in profitability dynamics and in the persistence of the underlying fundamentals that investors infer from observable accounting signals, consistent with the model's emphasis on firm-level heterogeneity in learning and mean reversion.

Panel A: Estimated profitability mean-reversion speed, λ_i.



Panel B: Estimated mean-reversion speed of the latent profitability driver, κ_i.

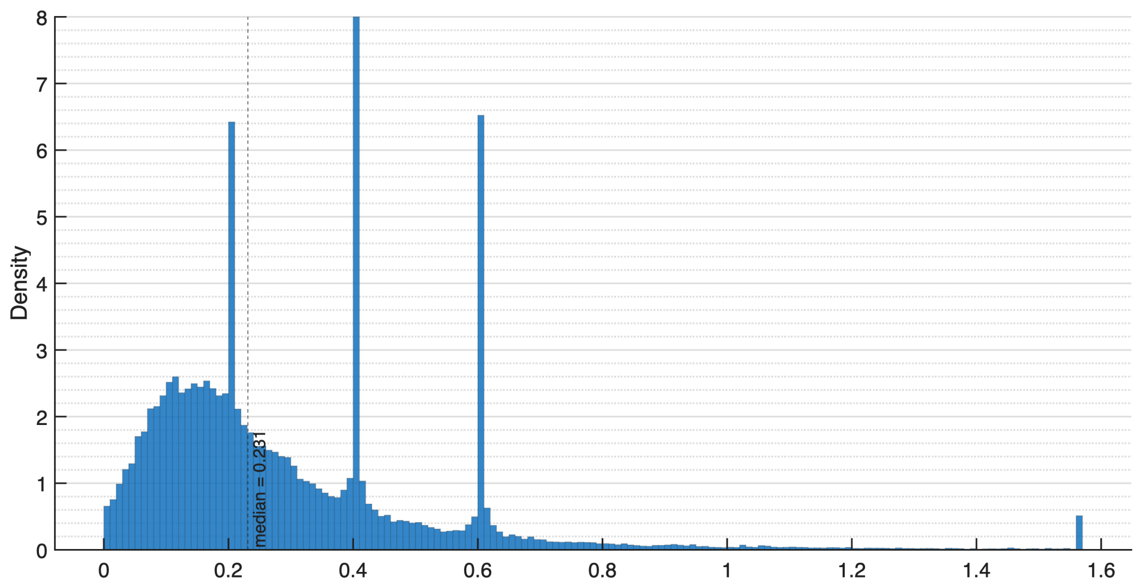


FIGURE 1: **The cross section of the estimated learning parameters.**
This figure reports the cross-sectional distributions of firm-level estimates of the profitability mean-reversion speed (λ_i; Panel A) and the mean-reversion speed of the unobserved profitability driver (κ_i; Panel B), obtained from our structural rolling-window estimation using firm and industry ROE data. We winsorize data at 1% tails. Initial values in the estimation are set to $\lambda_i^{(0)} = 0.5$ and $\kappa_i^{(0)} \in \{0.2, 0.4, 0.6\}$, with $\kappa_i^{(0)}$ chosen by firm size group.

4.4 Recovering beliefs and earnings surprises

We recover firm- and industry-level earnings surprises and update beliefs using a strict real-time convention. All objects are constructed at a monthly frequency (month-end t) using only information available as of that month-end. In particular, we take the most recently estimated learning parameters for firm i, denoted $\theta_{i,\tau(t)}$, where $\tau(t) \leq t$ is the latest parameter-estimation cutoff date for that firm; between re-estimation dates, we carry forward the most recent estimate.

Monthly observables and real-time signal availability. For each firm i, we construct a monthly panel indexed by month-end t. Firm-level ROE, X_{it}, is taken to be the most recently observed value available by month-end t (i.e., we carry forward the latest reported ROE until a new report arrives). In parallel, we recover the industry profitability signal each month. Specifically, for firm i's FF30 industry, we load the monthly industry benchmark series and align it to the firm's fiscal-quarter dates: at each month-end t, the industry signal s_{it} corresponds to the benchmark computed from the most recently completed fiscal quarter, using only firm-quarter ROE observations whose reporting dates are no later than t.

Recursive update and innovations. Given the monthly observables $\left(X_{it}, s_{it}\right)$ and the parameter vector $\theta_{i,\tau(t)}$, we apply Proposition 3.1 together with equations (9) and (10) recursively over time to update beliefs $\hat{\mu}_{it}$ and recover the model-implied innovations. In the implementation, we iterate over the full monthly history available for firm i up to month-end t using a fixed step size $dt = 1/4$ (consistent with quarterly scaling in the continuous-time formulation), and compute

$$d\widetilde{W}_{it}^{X} = \frac{\Delta X_{it} - \lambda_i\left(\hat{\mu}_{i,t-1} - X_{i,t-1}\right)dt}{\sigma_{x,i}}, \qquad d\widetilde{W}_{it}^{s} = \frac{\Delta s_{it} - \hat{\mu}_{i,t-1}dt}{\sigma_{s,i}},$$

where $\Delta X_{it} \equiv X_{it} - X_{i,t-1}$ and $\Delta s_{it} \equiv s_{it} - s_{i,t-1}$ denote month-to-month changes in the carriedforward firm ROE and the monthly industry signal, respectively. The belief update is then

$$\hat{\mu}_{it} = \hat{\mu}_{i,t-1} + \kappa_i\big(\bar{\mu}_i - \hat{\mu}_{i,t-1}\big)dt + \frac{\lambda_i \nu_{i,t-1}}{\sigma_{x,i}}\, d\widetilde{W}_{it}^X + \frac{\lambda_i \nu_{i,t-1}}{\sigma_{s,i}}\, d\widetilde{W}_{it}^s,$$

and the posterior variance is updated according to the discrete-time approximation to the Riccati equation,

$$\nu_{it} = \nu_{i,t-1} + \left(\sigma_{\mu,i}^2 - 2\kappa_i \nu_{i,t-1} - \frac{\lambda_i \nu_{i,t-1}^2}{\sigma_{x,i}^2} - \frac{\nu_{i,t-1}^2}{\sigma_{s,i}^2}\right)dt,$$

with the additional numerical safeguard that negative updates to ν_{it} are rejected (i.e., if $\nu_{it} < 0$ we keep $\nu_{it} = \nu_{i,t-1}$).

These real-time constructions are not only measurement choices but also deliver a sharp mechanism test: the learning model predicts that the return impact of a given innovation should scale with posterior uncertainty. Consistent with this prediction, Table VIII finds that the interaction terms $\nu_{it} d\widetilde{W}_{it}^x$ and $\nu_{it} d\widetilde{W}_{it}^s$ are positive and statistically significant in cross-sectional return regressions.

Initialization and signal volatilities. We initialize the recursion at the first available observation with $\hat{\mu}_{i0} = \bar{\mu}_{i,\tau(0)}$, where $\bar{\mu}_{i,\tau(0)}$ is the firm-specific long-run profitability estimate from the earliest available SMM window. We set the signal volatility $\sigma_{s,i}$ to the (annualized) standard deviation of the industry benchmark ROE series for firm i's industry.

Stored outputs. For each firm-month, we store the updated belief $\hat{\mu}_{it}$, the posterior variance ν_{it}, and both innovations $d\widetilde{W}_{it}^X$ and $d\widetilde{W}_{it}^s$, along with the parameter values used at that month-end (the most recent $\theta_{i,\tau(t)}$). Figure 2 reports the histogram of the estimated profitability innovations.

Appendix A provides additional details on the estimation procedure and summarizes the notation used throughout the paper.

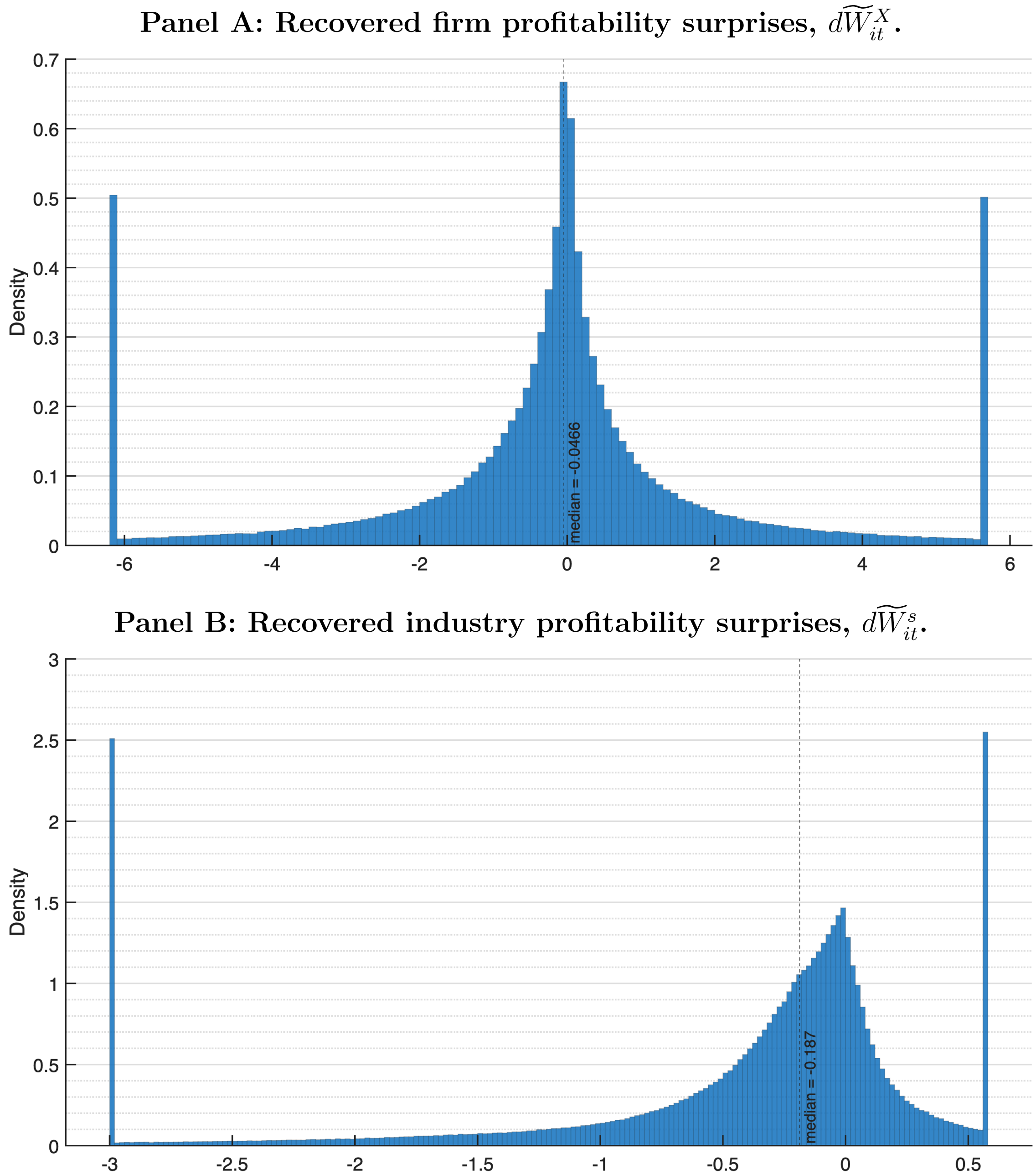


FIGURE 2: **Recovered profitability surprises.**
This figure plots histograms of the model-implied profitability surprise innovations recovered from our structural learning framework. Panel A shows firm-level profitability surprises, $d\widetilde{W}_{it}^{X}$, and Panel B shows surprises from the industry signal, $d\widetilde{W}_{it}^{s}$. The surprises are obtained from rolling-window estimation using only information available at each date. We winsorize the estimated surprises at the 1% tails.

5. Results

In this section, we document that the data line up closely with the core implications of our structural learning model. Firms with higher expected changes in profitability earn higher subsequent returns, with particularly strong and persistent spreads among small firms and for the long-run component of expected profitability. These patterns are monotonic across portfolios, survive a rich set of controls in Fama-MacBeth regressions, and remain visible over multi-month horizons, indicating that the expected profitability revision premium is a medium-run phenomenon rather than a short-lived anomaly. Finally, factor tests show that portfolios formed on expected profitability earn sizable alphas relative to standard factor models, suggesting that profitability learning captures a distinct and economically important dimension of discount-rate variation.

Our expected profitability revision measure is a structural expectation object implied by a learning framework and estimated on accounting data in real time. It is not a proxy that uses future realizations of profitability, and it does not rely on ex post information beyond the quarter in which beliefs are formed. It is also distinct from current profitability levels: it isolates belief revisions about a latent profitability driver and therefore need not be spanned by static profitability characteristics. Finally, while the measure is related to the information content of earnings, it is not mechanically equivalent to standard earnings-surprise or momentum constructs; instead it maps observable accounting signals into a disciplined estimate of expected profitability and its revision, which we then evaluate in return-predictability and factor-pricing tests.

5.1 Information set and real-time implementation

All empirical variables in this paper are constructed in real time using only information observable at the time beliefs are formed. Let $\mathcal{F}_{it}$ denote the information set available to investors for firm i at quarter t. By construction, $\mathcal{F}_{it}$ includes: (i) firm-level accounting information released up through quarter t that is needed to compute realized profitability X_{it}; (ii) industry-level accounting information released up through quarter t that is needed to compute the industry signal s_{it}; and (iii) the most recently estimated learning parameters obtained from a rolling window that ends at or before t. Therefore, the belief $\hat{\mu}_{it}$, expected profitability, and expected profitability revisions dated t are measurable with respect to $\mathcal{F}_{it}$.

When forming portfolios and running return predictability tests, we align signals dated t with subsequent returns so that the return window begins strictly after the signal is formed. This timing ensures that return predictability is not driven by look-ahead information.

5.2 Firm profitability dynamics

Understanding the dynamics of firm profitability expectations requires analyzing both short-term fluctuations and long-term trends. The estimated parameters λ_i, κ_i, together with the estimated earnings surprises, provide valuable insights into how firms adjust, revert, and learn about their profitability over time. We compute each firm's expected change in profitability using the estimated parameters λ_i and κ_i alongside the recovered $\hat{\mu}_{it}$. Applying Proposition 3.2 with a one-year horizon ($\tau = 1$), we estimate the next-year expected profitability, $E_t(X_{it+1})$.[2] We report summary statistics for all variables in Table I.

The mean expected change in firm profitability is 4.2% per annum, approximately one-third of the average firm profitability (ROE) of 11.25% observed in our sample. A significant portion of this expected change in profitability comes from the short-run component, which exhibits both positive and negative fluctuations. In contrast, the estimated long-run component remains positive, indicating a persistent underlying profitability trend. Firms vary in their speed of reversion to long-term profitability, their sensitivity to earnings surprises,

[2] Our results are robust to different τ values, as this parameter does not vary cross-sectionally. Changing τ alters the relative weights of current profitability and other factors but does not affect the overall conclusions.

and the efficiency with which investors update their beliefs. The considerable heterogeneity across these dimensions highlights the necessity of incorporating learning frictions and industry-wide information signals when evaluating firm profitability and its role in asset pricing.

Beyond averages, the cross-sectional dispersion in the learning parameters and belief variables is substantial and economically informative. Firms differ widely in how quickly their profitability responds to shocks (λ_i) and how fast the latent profitability driver reverts to its long-run level (κ_i). At one extreme, some firms exhibit almost no mean reversion in either dimension, so current profitability and beliefs drift for long periods without strong pull toward the anchor. At the other extreme, high-λ or high-κ firms adjust rapidly, implying that profitability surprises are absorbed into beliefs and discounted more quickly. This heterogeneity implies that the same earnings surprise can have very different implications for expected profitability across firms, depending on how tightly profitability is anchored to the long-run component.

On average, firm profitability exhibits a reversion speed of 0.46, though this varies significantly across firms, as indicated by a standard deviation of 0.39. Some firms show no mean reversion at all, while others revert rapidly. Closely linked is the rate of mean reversion of the profitability driver (κ_i). The average mean reversion rate is 0.23, also with a substantial degree of variation (0.22 standard deviation).

Firm-specific earnings shocks ($d\tilde{W}^x_{it}$) vary substantially across time and in the cross section, with a mean of -0.152 and a standard deviation of 2.5. By contrast, industry-wide earnings shocks ($d\tilde{W}^s_{it}$) tend to be more stable. With a mean of -0.44 and a standard deviation of 0.84, these shocks fluctuate within a much narrower band compared to firm-level surprises. The volatility of the firm's profitability driver (σ_{μ_i}) is quite low, averaging just 0.06. This estimated low volatility is due to the fact that we choose a reasonably stable source of information as an informative signal about the driver of firm profitability, which is the median ROE level within the industry.

The belief estimate of firm profitability ($\hat{\mu}_{it}$) is centered around 0.032, with limited dispersion (0.007 standard deviation). Most firms cluster within a reasonable range, though extreme cases show that some firms experience drastically negative or abnormally high expected profitability. In the long run, firms appear to stabilize around an estimated average profitability level ($\bar{\mu}_i$) of 0.1214, reinforcing the idea that profitability expectations converge toward a stable industry anchor.

5.3 Median splits

Each month, we sort firms into low (L) and high (H) portfolios by splitting the cross-section at the NYSE median of the expected change in profitability (or of its short-run and long-run components, respectively). In Table II, we show that firms with stronger expected profitability change are systematically different from firms with weaker expected profitability changes. Across all three panels, sorting on the total expected change in profitability, its short-run component, and its long-run component, high-expected-change firms (H) are smaller and younger than low-expected-change firms (L). Market size (ME) is markedly lower in the H portfolios, and firm age is lower by roughly 80 to 90 months. At the same time, high-expected-change firms exhibit lower current profitability (ROE), consistent with the idea that high expected changes reflect improvements from a depressed level of current earnings rather than already profitable firms.

Table II also shows that differences in expected profitability revisions line up systematically with firms' information environments and balance sheet strength. High-expected-change firms tend to have lower book leverage and higher forecast dispersion, consistent with a narrative in which investors face more uncertainty and disagreement about their future earnings paths. At the same time, these firms exhibit higher idiosyncratic risk and volatility (IdioRisk (Ali, Hwang, and Trombley, 2003), IdioVol3F (Ang, Hodrick, Xing, and Zhang, 2006)), suggesting that their cash flows and prices respond more strongly to firm-specific news. From a learning perspective, these are precisely the settings in which new information is both more informative and more consequential for beliefs, because investors have less precise priors and rely more heavily on earnings signals to refine their expectations.

The median splits on the short-run and long-run components echo this pattern. Firms in the high short-run-revision group often have depressed current profitability but stronger expected near-term improvements, consistent with the model's prediction that learning about $\hat{\mu}_{it}$ amplifies the impact of recent news when current conditions are weak. By contrast, firms in the high long-run-revision group are characterized by more gradual but persistent upgrades in beliefs about their long-term profitability. These firms tend to be small and young but not necessarily distressed; instead, they look like "emerging winners" whose profitability is expected to converge toward a more favorable long-run anchor. Taken together, the median-split evidence suggests that both short-horizon recovery and slow-moving trend re-assessment are important dimensions of the profitability learning channel.

5.4 Portfolio returns

Decile sorts. In Table III, we report average monthly excess returns and factor alphas for decile portfolios sorted on the expected change in profitability, with results shown for all firms and by size group (Micro, Small, Big).[3] Average returns rise broadly monotonically across deciles, and the high-minus-low ($H-L$) spread is economically and statistically meaningful: 0.35% per month (t = 2.84) across all firms, with larger spreads for Micro (0.61% per month, t = 3.57) and a still sizable spread for Small (0.31% per month, t = 2.70) and Big (0.32% per month, t = 2.44) firms. These spreads are robust to standard q-factor adjustments: the q-factor alpha of the $H-L$ portfolio is 0.38% per month (t = 3.33) for All firms, 0.46%

[3] We classify stocks as Micro, Small, and Big based on the NYSE 20th and 50th percentiles of market equity at June in year t.

(t = 3.08) for Micro, 0.03% (t = 0.28) for Small, and 0.39% (t = 3.11) for Big. The $q5$ alphas are similar in magnitude for All firms (0.42% per month, t = 3.37) and remain positive for Micro and Big firms. Moreover, the Gibbons–Ross–Shanken (GRS) test rejects the joint null of zero alphas for both the one-way deciles and the 3×10 size-by-decile test portfolios.

Decomposing the expected change in profitability into short- and long-run components shows that the short-run component delivers positive but more modest spreads (Table IV). In one-way decile sorts, average excess returns rise with the signal and the high-minus-low spread is $H-L$ = 0.28% per month (t = 2.38) for All firms. The spread is larger among Micro firms (0.42% per month), while it is weaker for Small firms (0.24% per month) and remains positive for Big firms (0.26% per month). Risk adjustment does not eliminate the pattern. Under the q-factor model, the $H-L$ alpha is 0.30% per month for All firms and 0.27% for Micro (t = 3.11), while remaining insignificant for Small and positive for Big (0.32%, t = 2.70); the GRS test rejects jointly zero alphas for the one-way deciles ($p_{GRS}^{1\times10}$ = 0.02) and for the 3×10 portfolios ($p_{GRS}^{3\times10}$ = 0.00). The $q5$ alphas are similar, with $H-L$ = 0.34% per month for All firms. By contrast, the long-run component (Table V) delivers larger and more robust spreads. In the raw decile sorts, the high-minus-low portfolio earns $H-L$ = 0.46% per month for All firms (t = 3.50), with spreads of 0.56% (t = 3.94) for Micro, 0.39% (t = 3.38) for Small, and 0.41% (t = 2.72) for Big firms.

The monotonicity of returns suggests that markets gradually reward incremental changes in expected profitability rather than only responding to extreme realizations. Importantly, the spreads remain large and statistically significant after controlling for the full set of q-factors, indicating that the expected profitability revision premium is not merely a reflection of known investment, size, or profitability effects embedded in those factor models. Instead, the results are consistent with expected profitability revisions capturing residual variation in discount rates that standard characteristics fail to span.

While both short-run and long-run revisions to expected profitability are priced, the more persistent long-run component earns the larger and more robust premium. Short-run revisions are naturally more volatile and occasionally reverse, leading to smaller and somewhat less stable H-L returns once standard factors are included. By contrast, portfolios sorted on the long-run component display stronger and more consistent spreads, particularly among Micro and Small firms, where learning frictions are most severe. This pattern is exactly what the model predicts: when investors are gradually revising their beliefs about the underlying latent driver of profitability, slow-moving trend upgrades represent more durable improvements in cash-flow expectations, which translate into higher discount rates and more persistent return premia.

The size breakdown further reinforces the state-dependence of the learning channel. For Micro and Small firms, both the total expected change and especially its long-run component generate large and precisely estimated spreads, whereas for Big firms the premiums, though still significant, are noticeably attenuated. This asymmetry is consistent with the idea that large firms operate in more transparent information environments, where beliefs about future profitability are already precise and earnings news has less scope to shift expectations. In contrast, for smaller firms with scarce analyst coverage and noisier signals, profitability revisions carry more incremental information, and the market appears to underreact more strongly, yielding larger learning premia.

5.5 Fama–MacBeth cross-sectional regressions

Table VI reports Fama–MacBeth regressions of one-month-ahead stock returns. The expected change in profitability is a highly significant predictor (t-statistic > 4) even after controlling for current ROE, accruals, size, BM, investment, and momentum. Firms with upward profitability revisions earn higher subsequent returns, providing further evidence of the expected profitability revision premium.

The cross-sectional regressions also help disentangle expected profitability revisions from correlated characteristics. Once we control for current ROE and accruals, the coefficient on the expected change in ROE remains large and highly significant, while the coefficients on the level of ROE are attenuated and in several specifications lose significance. This pattern is exactly what the learning model implies: what matters for discount rates is not whether a firm is currently profitable, but whether investors are revising upward or downward their beliefs about its future profitability path. Firms with high current profitability but little or no expected improvement do not command large premia, whereas firms starting from weaker profitability but with strong positive revisions do.

Table VII extends the analysis to multi-month horizons. Expected profitability revisions continue to forecast returns up to twelve months ahead, with large and significant coefficients. This horizon pattern is informative: under pure mean reversion in current profitability, we would expect short-lived predictability as earnings revert quickly, but limited power at longer horizons. The fact that revisions to expected profitability continue to matter at horizons up to twelve months suggests that investors are slowly updating beliefs about the latent profitability trend, consistent with gradual learning rather than mechanical reversal in cash flows.

Economically, the regression slopes imply that a move from a low to a high expected-revision firm translates into meaningful differences in subsequent returns, even after controlling for size, value, investment, and momentum. In other words, the model-implied belief updates capture a dimension of variation in expected returns that is not easily proxied by standard characteristics. This supports interpreting the expected profitability revision premium as a distinct pricing channel rather than a re-labeling of existing effects.

5.6 Testing the learning channel

In Table VIII, we directly test the mechanism emphasized by the model, namely, that it is the interaction of earnings news with belief uncertainty that should be priced. Consistent with this prediction, firm-level surprises are most strongly associated with returns precisely when posterior variance is high, and the incremental explanatory power of these interaction terms is sizable. Industry-level surprises scaled by uncertainty do not seem to predict future returns.

While Table VI establishes that expected profitability revisions predict returns, Table VIII goes one step deeper by testing the learning channel directly. The learning model implies that belief revisions are proportional to posterior uncertainty, so the same earnings surprise should move prices more when ν_{it} is high. In the data, we find that $\nu_{it} d\widetilde{W}_{it}^{x}$ command positive and statistically significant premia, even after controlling for profitability levels and firm characteristics, supporting the interpretation that our expected-profitability revisions reflect economically meaningful learning about firm fundamentals.

5.7 The expected profitability revision factor

Each month t, stocks are sorted into expected-change-in-ROE groups (robust, average, weak) based on 30-40-30 NYSE breakpoints. Expected change in ROE is measured using the most recent return on equity (ROE) data, as of month t. Stocks are also divided into two size groups (small, large) using the median NYSE breakpoint. These groups are rebalanced monthly. Portfolios are rebalanced monthly. Our results are robust to an alternative three-way factor construction that additionally sorts on the investment-to-assets ratio (I/A).

We form six portfolios from the intersection of the two size groups and three expected-change-in-ROE groups. Each portfolio's value-weighted average return is calculated for month $t + 1$. The two-way expected profitability factor is the average return on robust portfolios minus the average return on weak portfolios:

$$\Delta\text{ExpProf}_t = \frac{R_{t,t+1}^{Robust,Large} + R_{t,t+1}^{Robust,Small} - R_{t,t+1}^{Weak,Large} - R_{t,t+1}^{Weak,Small}}{2}. \tag{14}$$

Table IX presents time-series spanning regressions testing whether expected changes in profitability earn abnormal returns unexplained by standard factor models. The average return spread is 0.27% per month. This return spread persist after controlling for Fama-French and q-factor models, suggesting that profitability revisions capture information beyond existing factors.

Alpha estimates remain statistically significant across specifications, with t-statistics often above 3.0 or 4.0. Even in the six-factor Fama-French model, the alpha of the expected profitability factor remains high (0.22% per month, t-statistic = 4.05). Both short- and long-run components deliver significant alphas: the short-run component generates 0.20% per month in FF6 regressions, while the long-run component delivers 0.16%. These results show that both dimensions of expected profitability revisions contribute to return predictability.

5.8 Bayesian factor selection test

We evaluate whether an expected profitability revision factor is selected in Bayesian model comparison. Specifically, we implement Bayesian factor selection tests to quantify which subset of candidate factors is most likely to be priced, conditional on always including the market factor, following the model-comparison logic in Chib et al. (2020) and the factor-selection implementation based on marginal likelihoods in Barillas and Shanken (2018). Concretely, we stack monthly excess returns for $K = 11$ factors into the matrix F (market plus ten non-market candidates: SMB, HML, RMW, CMA, Mom, R_ME, R_IA, R_ROE, R_EG, and ΔExpProf). We then enumerate a universe of models that differ by which of the $K - 1$ non-market factors are treated as risk factors, compute each model's marginal likelihood, and convert these into posterior model probabilities. The "best" model is the one with the highest posterior probability, and its reported risk-factor pattern (excluding the market) indicates which factors are selected as priced in that model.

Beyond identifying the single best model, we summarize evidence across the entire model space via posterior inclusion probabilities For each factor, the inclusion probability is the posterior probability that the factor belongs to the priced set, obtained by summing posterior probabilities of all models in which that factor is included. We compute these both in the full sample (yielding "cumulative" inclusion probabilities) and recursively over time: at each month t (after an initial window of $minT$ = 120 months), we re-estimate model posteriors using data $F_{1:t}$ and track (i) posterior probabilities for the top models and (ii) time-varying inclusion probabilities for each non-market factor. This recursive procedure produces a dynamic view of factor importance, showing when particular factors become more/less likely to be priced, while the full-sample results highlight that R_{IA}, R_{EG}, and ΔExpProf receive the strongest joint support as non-market risk factors in the maximum-posterior specification.

Figure 3 plots the full-sample posterior inclusion probabilities for each non-market factor (conditional on always including the market). Two factors stand out as essentially certain components of the priced factor set: R_EG and ΔExpProf both have inclusion probabilities of 1.00. Among the remaining candidates, R_IA receives very strong support with an inclusion probability of roughly 0.89 and SMB at about 0.45. The evidence for the other factors is notably weaker. Overall, the bar chart indicates that the posterior mass concentrates on models that almost surely include R_EG and ΔExpProf, frequently include R_IA, and treat the remaining factors as comparatively less relevant.

The bottom panel of Figure 4 plots the Chib et al. (2020) recursive posterior inclusion probabilities for each non-market factor, computed each month using data up to that date (after an initial 10-year burn-in). Two factors emerge as persistently central in the post-2000 sample: R_EG and ΔExpProf rise to (and then remain essentially at) inclusion probability

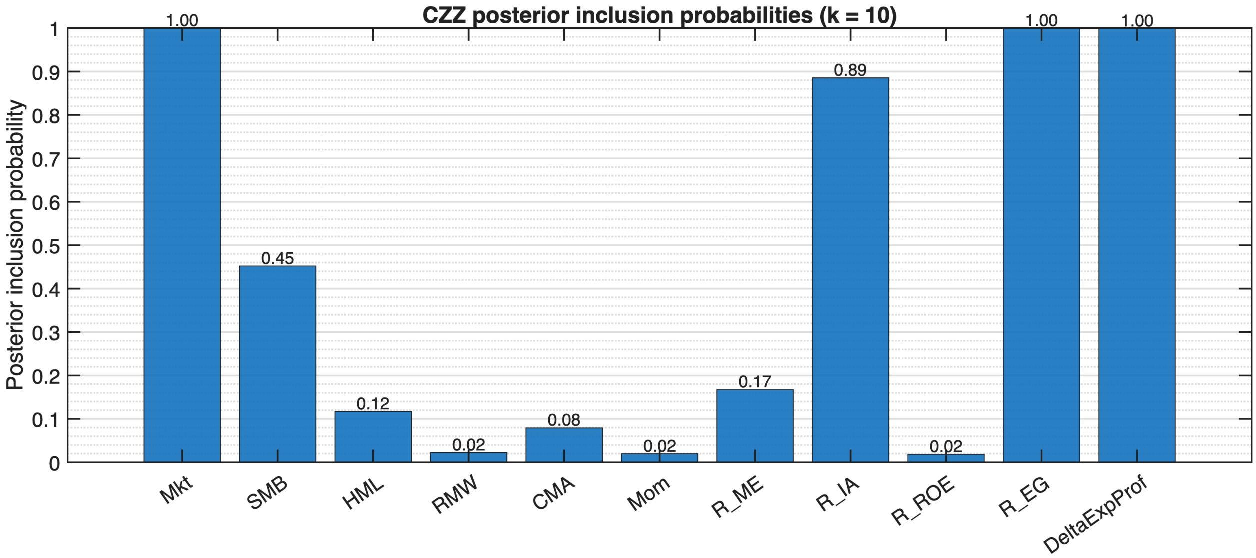


Figure 3: **Posterior factor inclusion probabilities (Chib et al., 2020).**
This figure plots full-sample posterior risk-factor inclusion probabilities from the (Chib et al., 2020)'s model-comparison procedure. The market factor (Mkt) is included by construction in every candidate model. For each remaining factor, the height of the bar equals the posterior probability that the factor is priced (i.e., classified as a risk factor) after integrating over the entire model space: it is computed by summing posterior model probabilities across all models in which that factor is included in the priced-factor set.

one, indicating that the posterior mass concentrates on models that almost surely include these two factors once sufficient data accumulate. In contrast, several classic and q-style factors display time-varying but generally weaker support. SMB starts with moderate inclusion probabilities in the early 1980s but declines markedly by the late 1980s and stays low thereafter. Momentum spikes to near certainty in the late 1980s, then collapses quickly in the early 1990s and remains close to zero for most of the post-2000 period. Finally, R_IA exhibits episodic importance: it rises materially around the late 1990s/early 2000s, falls back in the mid-2000s, and then re-emerges with noticeably higher inclusion probabilities in the 2010s and especially near the end of the sample.

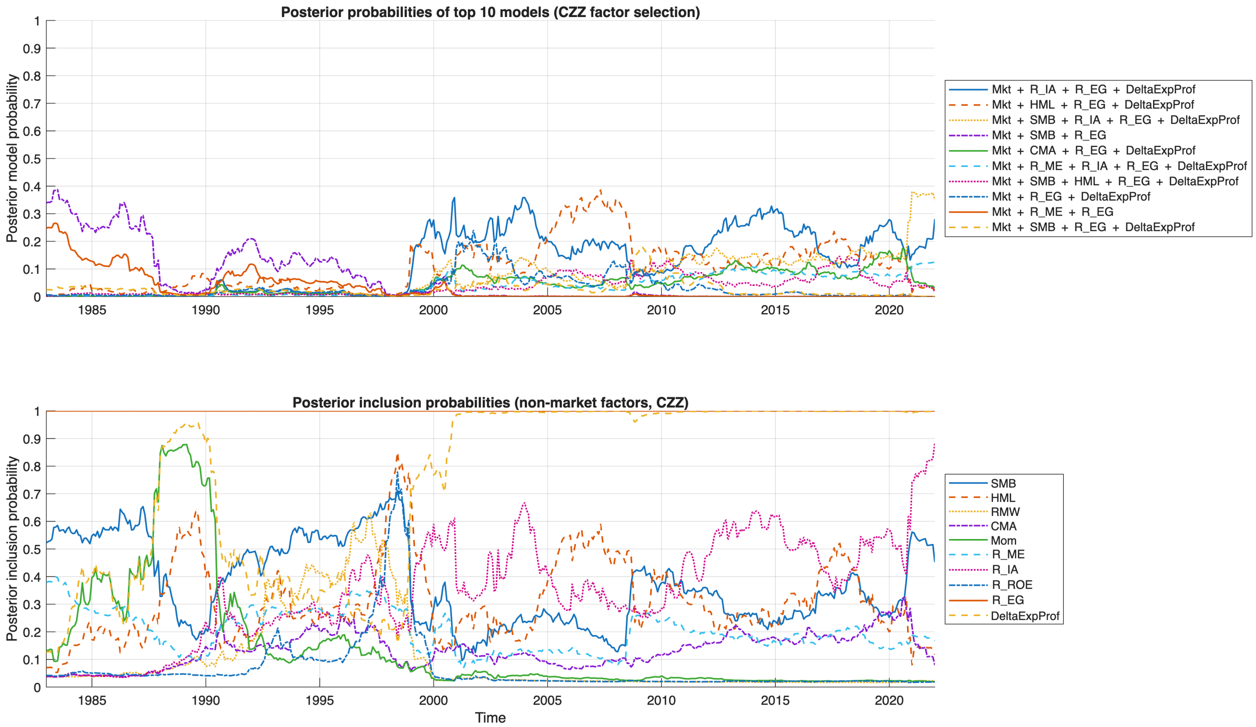


FIGURE 4: **Time-varying posterior inclusion probabilities.**
The figure plots recursive posterior inclusion probabilities for each non-market factor under the Chib et al. (2020) model-scan procedure. At each month t (after an initial burn-in window), posterior model probabilities are re-computed using factor returns observed up to t, and each factor's inclusion probability is obtained by summing posterior probabilities across all models that include that factor. The paths highlight that R_EG and ΔExpProf become (and remain) near-certain inclusions in the priced factor set in the post-2000 sample, while other factors exhibit weaker and more episodic posterior inclusion.

6. Conclusion

This paper develops and implements a theory-consistent proxy for expected profitability revisions, defined as the expected change in firm profitability relative to its current level, using a structural learning framework in which investors update beliefs about a latent profitability component from firm earnings and an industry profitability signal. The resulting measure is estimated from accounting data in rolling windows to mitigate look-ahead bias and yields a transparent decomposition into a short-run and a long-run anchor component.

Empirically, expected profitability revisions forecast returns in the cross-section and in portfolio sorts. Portfolios sorted on expected profitability change exhibit monotonic return spreads that are economically meaningful, with the largest premia concentrated among micro-cap and small firms. Decomposing revisions shows that the long-run component delivers the most robust premia in decile sorts, consistent with persistent belief updates being more strongly priced, while the short-run component is positive but more volatile and less stable across subsamples. In time-series factor tests, a traded expected profitability change factor earns a sizable and statistically significant premium and remains strongly significant relative to Fama–French and q-factor benchmarks. Head-to-head spanning tests indicate that this factor is complementary rather than fully subsuming existing factor structures, and Bayesian model comparison assigns high posterior inclusion probabilities to the expected profitability change factor alongside a small set of other factors.

Beyond documenting an expected profitability revision premium, we provide direct evidence for the underlying learning mechanism documenting that uncertainty-scaled surprises from firm earnings and industry signals are priced in the cross section, consistent with Bayesian belief updating about a latent profitability driver.

Overall, the evidence suggests that aligning measurement with theory by moving from current profitability levels to expected profitability revisions implied by an explicit learning process sharpens profitability-based return predictability and clarifies how earnings information is incorporated into prices, particularly in settings with greater information frictions.

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Tables

Table I: Summary statistics. (1973-2021)

This table reports descriptive statistics for firm characteristics, profitability variables, and estimated learning parameters. Variables include ROE, accruals, size (ME), book-to-market (BM), investment, returns, and momentum, as well as model parameters (λ_i, κ_i, $\sigma_{\mu i}$, ν_i, $\hat{\mu}_i$, $\bar{\mu}_i$). The table provides an overview of the distributional properties of inputs variables and expected profitability outputs in the structural learning framework. Data comes from Chen and Zimmermann (2022).

	Mean	StdDev	Min	Max	25th	75th
ROE	0.1125	2.1848	-33.3330	31.5830	0.0429	0.6734
Accruals	-0.0298	0.0839	-0.4404	0.4060	-0.0688	0.0083
ME	3.45E+06	2.22E+07	8.83E+01	2.90E+09	6.05E+04	1.23E+06
BM	0.8022	0.7436	-1.5375	10.2630	0.3494	1.0337
Investment	0.9964	0.5009	0.0071	5.3760	0.6963	1.2021
ret	0.0137	0.1384	-0.7161	1.6356	-0.0562	0.0721
Mom12m	0.1608	0.5686	-0.9432	12.9790	-0.1484	0.3456
λ_i	0.4588	0.3900	0.0098	1.7641	0.1608	0.5931
κ_i	0.2279	0.2224	0.0055	1.1934	0.0764	0.3023
$d\tilde{W}^x_{it}$	-0.1522	2.4791	-6.1488	5.6135	-0.9205	0.6447
$d\tilde{W}^s_{it}$	-0.4425	0.8354	-2.9903	0.5710	-0.5753	0.0117
σ_{μ_i}	0.0608	0.0636	0.0000	0.6356	0.0227	0.0701
ν_{it}	1.37E-05	1.27E-05	2.86E-07	7.26E-05	5.33E-06	1.79E-05
$\hat{\mu}_{it}$	0.0324	0.0072	0.0159	0.0523	0.0278	0.0370
$\bar{\nu}_i$	0.0004	0.0008	0.0000	0.0418	0.0001	0.0004
$\bar{\mu}_i$	0.1214	0.1908	-0.7662	0.5382	0.0671	0.2205
Expected change in ROE	0.0423	0.0296	0.0000	0.2807	0.0180	0.0595
... over short run	0.0368	0.0257	0.0000	0.2679	0.0163	0.0515
... over long run	0.0055	0.0075	0.0000	0.0873	0.0008	0.0070

TABLE II: Average Firm Characteristics for Portfolios Sorted on the Expected Change in Profitability (1973-2021)

This table reports average firm characteristics for portfolios formed each month by splitting firms into low (L) and high (H) portfolios based on whether their expected change in profitability is below or above the cross-sectional median. For each month, portfolio-level characteristics are computed as value-weighted averages of firm-level variables, and the time-series average of these monthly portfolio characteristics is reported in the table. The column $H-L$ shows the difference in average characteristics between the high and low portfolios, and t reports the Newey–West t-statistic for this monthly high-minus-low difference. Data on firm-level signals comes from Chen and Zimmermann (2022).

	Expected Change in Profitability				Short-run Component				Long-run Component			
	L	H	$H-L$	t	L	H	$H-L$	t	L	H	$H-L$	t
ME	4.31E+06	1.84E+06	-2.46E+06	-[7.77]	4.19E+06	1.98E+06	-2.21E+06	-[8.12]	4.75E+06	1.39E+06	-3.36E+06	-[6.77]
ROE	0.25	0.06	-0.19	-[9.73]	0.24	0.07	-0.17	-[10.08]	0.28	0.04	-0.24	-[9.95]
Accruals	0.03	0.03	0.00	[3.83]	0.03	0.03	0.00	[2.88]	0.03	0.03	0.00	[5.61]
Beta	0.91	1.15	0.24	[19.38]	0.91	1.14	0.23	[19.76]	0.92	1.14	0.22	[12.12]
BM	0.86	0.87	0.02	[0.98]	0.86	0.87	0.01	[0.50]	0.84	0.90	0.06	[4.21]
Book Leverage	5.21	3.68	-1.53	-[2.25]	5.21	3.68	-1.52	-[2.15]	5.05	3.84	-1.22	-[1.83]
Coskewness	0.21	0.21	0.00	-[0.33]	0.21	0.21	0.00	[0.13]	0.21	0.20	-0.01	-[1.03]
Debt Issuance	0.57	0.54	-0.03	-[4.72]	0.56	0.54	-0.03	-[4.14]	0.57	0.53	-0.05	-[5.58]
Equity Duration	15.46	15.99	0.53	[7.77]	15.48	15.98	0.50	[7.73]	15.52	15.93	0.41	[5.18]
Firm Age	349.97	264.36	-85.61	-[12.49]	348.34	268.53	-79.81	-[12.60]	351.59	258.05	-93.55	-[11.35]
FirmAgeMom	0.08	0.14	0.04	[3.96]	0.09	0.13	0.03	[2.82]	0.09	0.13	0.04	[3.17]
Forecast Dispersion	0.16	0.24	0.08	[10.68]	0.16	0.23	0.07	[10.34]	0.15	0.25	0.10	[13.04]
IdioRisk	0.02	0.03	0.01	[12.98]	0.02	0.03	0.01	[12.44]	0.02	0.03	0.01	[13.65]
IdioVol3F	0.02	0.03	0.01	[13.49]	0.02	0.03	0.01	[12.94]	0.02	0.03	0.01	[13.92]
Illiquidity	0.00	0.00	0.00	[5.56]	0.00	0.00	0.00	[5.41]	0.00	0.00	0.00	[5.62]
IndMom	0.08	0.09	0.01	[4.15]	0.08	0.09	0.01	[4.62]	0.08	0.09	0.00	[2.49]
IndRetBig	0.02	0.02	0.00	[4.61]	0.02	0.02	0.00	[4.65]	0.02	0.02	0.00	[3.64]
Investment	1.00	0.99	0.00	-[1.04]	1.00	1.00	0.00	-[0.56]	1.00	0.99	-0.01	-[3.04]
Leverage	2.76	2.43	-0.34	-[1.42]	2.86	2.33	-0.53	-[2.30]	2.52	2.67	0.15	[0.77]
Mom12m	0.14	0.18	0.04	[3.78]	0.14	0.18	0.04	[4.11]	0.15	0.16	0.01	[1.45]
MomRev	0.51	0.56	0.05	[3.43]	0.52	0.56	0.04	[2.59]	0.50	0.57	0.07	[6.22]
RD	0.05	0.07	0.01	[6.38]	0.06	0.07	0.01	[6.23]	0.05	0.07	0.02	[6.52]
Volume to market equity	0.11	0.14	0.03	[6.86]	0.11	0.14	0.03	[6.33]	0.11	0.13	0.02	[6.30]

Table III: Expected Change in Profitability Decile Portfolios (1973-2021)

Breakpoints for portfolio assignment are computed using NYSE firms only, and returns within each decile are value-weighted. For each decile portfolio, the table reports average excess return $\bar{R}$, the q-factor alpha α_q, and the $q5$ alpha α_{q5}. p_{GRS} is the p-value of the Gibbons–Ross–Shanken test of the null that decile alphas are jointly zero. For two-way sorted portfolios, p_{GRS} is the corresponding p-value for the 3×10 testing portfolios. The "All" rows are from one-way decile sorts. The t-values shown in brackets are adjusted for heteroscedasticity and autocorrelation.

	L	2	3	4	5	6	7	8	9	H	$H-L$
						$\bar{R}$					
All	0.57	0.65	0.52	0.75	0.60	0.67	0.71	0.72	0.78	0.92	0.35
	[2.79]	[3.49]	[2.75]	[3.74]	[3.02]	[2.94]	[3.19]	[3.21]	[3.22]	[3.68]	[2.84]
Micro	0.95	0.88	0.98	1.12	1.02	1.22	1.14	1.32	1.37	1.56	0.61
	[3.22]	[3.36]	[3.11]	[3.88]	[3.52]	[4.04]	[3.85]	[4.11]	[4.14]	[4.39]	[3.57]
Small	0.83	0.79	0.83	0.88	0.97	0.99	0.96	1.00	1.14	1.14	0.31
	[2.95]	[3.37]	[3.46]	[3.66]	[3.76]	[3.86]	[3.62]	[3.71]	[4.06]	[3.90]	[2.70]
Big	0.56	0.65	0.51	0.75	0.57	0.62	0.68	0.66	0.76	0.88	0.32
	[2.73]	[3.40]	[2.67]	[3.64]	[2.84]	[2.68]	[3.05]	[2.97]	[3.11]	[3.51]	[2.44]
						α_q ($p_{GRS}^{1\times10}=0.01$, $p_{GRS}^{3\times10}=0.00$)					
All	-0.07	0.03	-0.03	0.12	-0.02	0.10	-0.02	0.06	0.21	0.32	0.38
	-[1.29]	[0.41]	-[0.57]	[1.71]	-[0.24]	[1.13]	-[0.23]	[0.87]	[2.93]	[3.38]	[3.33]
Micro	0.27	0.04	-0.03	0.36	0.23	0.36	0.28	0.63	0.54	0.73	0.46
	[2.41]	[0.30]	-[0.17]	[3.69]	[2.59]	[4.04]	[3.06]	[6.96]	[5.39]	[5.76]	[3.08]
Small	0.23	-0.07	-0.04	0.04	0.11	0.16	0.09	0.21	0.36	0.26	0.03
	[2.90]	-[0.68]	-[0.53]	[0.47]	[1.37]	[2.06]	[1.15]	[2.91]	[4.33]	[2.84]	[0.28]
Big	-0.08	0.03	-0.03	0.12	-0.03	0.08	-0.02	0.03	0.21	0.32	0.39
	-[1.39]	[0.46]	-[0.52]	[1.62]	-[0.42]	[0.81]	-[0.28]	[0.35]	[2.72]	[3.06]	[3.11]
						α_{q5} ($p_{GRS}^{1\times10}=0.01$, $p_{GRS}^{3\times10}=0.00$)					
All	-0.04	-0.01	-0.06	0.06	-0.03	0.07	-0.01	0.16	0.34	0.38	0.42
	-[0.70]	-[0.09]	-[0.85]	[0.86]	-[0.44]	[0.80]	-[0.13]	[2.01]	[4.59]	[3.75]	[3.37]
Micro	0.21	0.09	0.17	0.33	0.18	0.41	0.29	0.60	0.57	0.66	0.45
	[1.77]	[0.74]	[0.83]	[3.10]	[1.82]	[4.35]	[2.91]	[6.13]	[5.29]	[4.84]	[2.78]
Small	0.29	-0.04	-0.06	0.07	0.14	0.11	0.10	0.21	0.45	0.32	0.03
	[3.33]	-[0.39]	-[0.69]	[0.82]	[1.58]	[1.30]	[1.22]	[2.67]	[5.07]	[3.16]	[0.25]
Big	-0.05	-0.01	-0.06	0.06	-0.04	0.06	-0.01	0.14	0.36	0.40	0.44
	-[0.81]	-[0.09]	-[0.85]	[0.76]	-[0.55]	[0.56]	-[0.12]	[1.52]	[4.31]	[3.54]	[3.26]

Table IV: Short-Run Component: Expected Change in Profitability Decile Portfolios (1973-2021)

Breakpoints for portfolio assignment are computed using NYSE firms only, and returns within each decile are value-weighted. For each decile portfolio, the table reports average excess return $\bar{R}$, the q-factor alpha α_q, and the $q5$ alpha α_{q5}. p_{GRS} is the p-value of the Gibbons–Ross–Shanken test on the null that decile alphas are jointly zero. For two-way sorted portfolios, p_{GRS} is the corresponding p-value for the 3×10 testing portfolios. The "All" rows are from one-way decile sorts. The t-values shown in brackets are adjusted for heteroscedasticity and autocorrelations.

	L	2	3	4	5	6	7	8	9	H	$H-L$
						$\bar{R}$					
All	0.57	0.63	0.54	0.67	0.69	0.69	0.70	0.85	0.75	0.85	0.28
	[2.80]	[3.37]	[2.79]	[3.34]	[3.42]	[3.17]	[3.15]	[3.66]	[3.15]	[3.42]	[2.38]
Micro	0.96	0.75	1.16	1.11	1.02	1.21	1.23	1.24	1.51	1.39	0.42
	[3.25]	[2.99]	[3.22]	[3.75]	[3.64]	[4.07]	[4.00]	[4.02]	[4.40]	[3.93]	[2.60]
Small	0.83	0.82	0.83	0.94	0.99	0.96	1.05	1.03	1.09	1.08	0.24
	[3.14]	[3.66]	[3.65]	[4.09]	[4.14]	[3.94]	[4.18]	[4.00]	[4.02]	[3.83]	[2.19]
Big	0.56	0.63	0.53	0.65	0.66	0.65	0.65	0.83	0.72	0.82	0.26
	[2.74]	[3.29]	[2.69]	[3.15]	[3.26]	[2.92]	[2.93]	[3.52]	[3.06]	[3.30]	[2.11]
						α_q ($p^{1\times10}_{GRS}$ = 0.02, $p^{3\times10}_{GRS}$ = 0.00)					
All	-0.06	-0.02	-0.03	0.07	0.07	0.15	-0.03	0.13	0.17	0.25	0.30
	-[1.09]	-[0.24]	-[0.47]	[1.07]	[1.01]	[1.88]	-[0.40]	[1.74]	[2.44]	[2.80]	[2.78]
Micro	0.28	-0.12	0.19	0.34	0.20	0.40	0.42	0.44	0.72	0.55	0.27
	[2.52]	-[0.89]	[1.00]	[3.33]	[2.11]	[4.54]	[4.58]	[4.80]	[7.36]	[4.24]	[1.79]
Small	0.22	-0.04	-0.06	0.09	0.10	0.14	0.23	0.16	0.27	0.20	-0.02
	[2.78]	-[0.44]	-[0.69]	[1.09]	[1.28]	[2.00]	[3.18]	[2.34]	[3.19]	[2.21]	-[0.18]
Big	-0.06	-0.01	-0.03	0.06	0.06	0.13	-0.06	0.12	0.18	0.25	0.32
	-[1.18]	-[0.19]	-[0.46]	[0.85]	[0.85]	[1.52]	-[0.76]	[1.44]	[2.28]	[2.66]	[2.70]
						α_{q5} ($p^{1\times10}_{GRS}$ = 0.04, $p^{3\times10}_{GRS}$ = 0.04)					
All	-0.02	-0.03	-0.06	0.06	0.04	0.14	-0.04	0.20	0.28	0.32	0.34
	-[0.45]	-[0.46]	-[0.93]	[0.78]	[0.52]	[1.64]	-[0.55]	[2.56]	[3.86]	[3.33]	[2.90]
Micro	0.22	-0.09	0.46	0.27	0.19	0.37	0.49	0.42	0.77	0.48	0.26
	[1.88]	-[0.63]	[2.30]	[2.45]	[1.92]	[3.82]	[4.90]	[4.21]	[7.26]	[3.47]	[1.60]
Small	0.28	-0.03	-0.05	0.11	0.13	0.09	0.27	0.14	0.36	0.26	-0.02
	[3.24]	-[0.32]	-[0.52]	[1.22]	[1.52]	[1.21]	[3.36]	[1.89]	[3.99]	[2.66]	-[0.14]
Big	-0.03	-0.03	-0.07	0.05	0.03	0.13	-0.07	0.21	0.30	0.34	0.37
	-[0.54]	-[0.44]	-[0.99]	[0.62]	[0.39]	[1.42]	-[0.87]	[2.29]	[3.59]	[3.31]	[2.92]

Table V: Long-Run Component: Expected Change in Profitability Decile Portfolios (1973-2021)

Breakpoints for portfolio assignment are computed using NYSE firms only, and returns within each decile are value-weighted. For each decile portfolio, the table reports average excess return $\bar{R}$, the q-factor alpha α_q, and the $q5$ alpha α_{q5}. p_{GRS} is the p-value of the Gibbons–Ross–Shanken test on the null that decile alphas are jointly zero. For two-way sorted portfolios, p_{GRS} is the corresponding p-value for the 3×10 testing portfolios. The "All" rows are from one-way decile sorts. The t-values shown in brackets are adjusted for heteroscedasticity and autocorrelations.

	L	2	3	4	5	6	7	8	9	H	$H-L$
						$\bar{R}$					
All	0.63	0.61	0.67	0.64	0.71	0.56	0.67	0.56	0.79	1.09	0.46
	[3.01]	[3.21]	[3.45]	[3.36]	[3.49]	[2.67]	[3.04]	[2.47]	[3.30]	[4.07]	[3.50]
Micro	0.96	1.01	0.99	1.07	1.23	1.11	1.09	1.24	1.37	1.52	0.56
	[3.15]	[3.79]	[3.52]	[3.63]	[3.67]	[3.81]	[3.58]	[3.88]	[4.01]	[4.62]	[3.94]
Small	0.88	0.82	0.85	0.88	0.95	0.94	0.85	0.89	1.08	1.27	0.39
	[3.22]	[3.51]	[3.55]	[3.76]	[4.03]	[3.80]	[3.39]	[3.41]	[4.13]	[4.85]	[3.38]
Big	0.62	0.60	0.67	0.63	0.68	0.52	0.63	0.50	0.76	1.03	0.41
	[2.95]	[3.11]	[3.37]	[3.27]	[3.34]	[2.48]	[2.87]	[2.23]	[3.18]	[3.73]	[2.72]
						α_q ($p^{1\times10}_{GRS}$ = 0.00, $p^{3\times10}_{GRS}$ = 0.00)					
All	0.04	-0.09	0.09	0.04	0.07	-0.14	0.04	-0.16	0.15	0.65	0.61
	[0.86]	-[1.50]	[1.52]	[0.58]	[1.13]	-[2.23]	[0.54]	-[2.12]	[1.86]	[7.16]	[5.49]
Micro	0.28	0.20	0.18	0.20	0.18	0.32	0.29	0.45	0.61	0.70	0.42
	[2.48]	[1.78]	[1.58]	[1.93]	[0.94]	[3.20]	[3.06]	[4.84]	[5.59]	[7.60]	[3.38]
Small	0.36	-0.08	-0.08	-0.05	0.06	0.06	0.04	0.09	0.29	0.51	0.15
	[4.29]	-[0.85]	-[0.92]	-[0.61]	[0.82]	[0.82]	[0.56]	[1.13]	[3.78]	[6.20]	[1.32]
Big	0.03	-0.09	0.10	0.05	0.07	-0.17	0.03	-0.19	0.15	0.71	0.67
	[0.66]	-[1.45]	[1.60]	[0.68]	[1.04]	-[2.41]	[0.33]	-[2.23]	[1.63]	[6.15]	[5.00]
						α_{q5} ($p^{1\times10}_{GRS}$ = 0.00, $p^{3\times10}_{GRS}$ = 0.00)					
All	0.07	-0.12	0.03	-0.01	0.09	-0.08	0.09	-0.02	0.20	0.73	0.66
	[1.33]	-[1.86]	[0.46]	-[0.19]	[1.25]	-[1.22]	[1.19]	-[0.25]	[2.27]	[7.48]	[5.53]
Micro	0.22	0.17	0.24	0.21	0.44	0.31	0.28	0.40	0.56	0.70	0.48
	[1.80]	[1.38]	[1.97]	[1.86]	[2.15]	[2.89]	[2.78]	[4.00]	[4.77]	[7.00]	[3.56]
Small	0.40	-0.07	-0.02	-0.02	0.08	0.05	0.06	0.15	0.30	0.55	0.15
	[4.47]	-[0.73]	-[0.20]	-[0.17]	[0.89]	[0.63]	[0.78]	[1.76]	[3.64]	[6.29]	[1.25]
Big	0.07	-0.12	0.03	-0.01	0.08	-0.10	0.09	-0.04	0.21	0.81	0.74
	[1.15]	-[1.78]	[0.48]	-[0.11]	[1.12]	-[1.30]	[0.99]	-[0.44]	[2.12]	[6.58]	[5.16]

Table VI: Fama–MacBeth regressions. (1973-2021)

Cross-sectional regression results of next-month stock returns being regressed on firm characteristics. Expected change in profitability strongly predicts returns after controlling for ROE, accruals, size (ME), BM, investment, and past returns. Newey-West adjusted t-stats are in parentheses. All right-hand variables are Winsorized at 1% level.

	$r_{t,t+1}$	$r_{t,t+1}$	$r_{t,t+1}$	$r_{t,t+1}$	$r_{t,t+1}$
Intercept	0.01	0.01	0.01	0.01	0.01
	[4.90]	[4.10]	[4.20]	[3.87]	[4.03]
Expected change in ROE		0.08		0.09	
		[6.14]		[5.63]	
Short-horizon			0.04		0.04
			[2.28]		[2.39]
Long-horizon			0.38		0.37
			[8.47]		[8.07]
ROE	0.02	0.03	0.03		
	[4.07]	[4.30]	[4.46]		
Accruals	0.02	0.02	0.02		
	[6.58]	[6.47]	[6.34]		
ME	0.00	0.00	0.00		
	-[2.53]	-[0.96]	-[0.95]		
BM	0.00	0.00	0.00		
	[6.58]	[6.65]	[6.50]		
Investment	0.00	0.00	0.00		
	[5.94]	[5.15]	[5.09]		
ret(-1,0)	-0.05	-0.05	-0.05		
	-[10.97]	-[11.23]	-[11.34]		
ret(-12,-1)	0.00	0.00	0.00		
	[2.38]	[2.28]	[2.30]		
Adj R2	3.27%	3.55%	3.67%	0.36%	0.49%
Obs.	1,404,742	1,380,117	1,380,117	1,664,057	1,664,057
Controls	YES	YES	YES	NO	NO

Table VII: Fama–MacBeth regressions: Longer-horizons. (1973-2021)

Cross-sectional regressions of cumulative returns over 3-, 6-, 9-, and 12-month horizons being regressed on firm characteristics. Expected change in ROE predicts returns up to nine months ahead, while current ROE loses predictive power beyond six months ahead. Newey-West adjusted t-stats are in parentheses. All right-hand variables are Winsorized at 1% level.

	$r_{t,t+3}$	$r_{t,t+6}$	$r_{t,t+9}$	$r_{t,t+12}$
Intercept	0.03	0.06	0.10	0.13
	[4.41]	[4.95]	[5.82]	[6.78]
ROE	0.06	0.09	0.07	0.06
	[3.42]	[2.80]	[1.69]	[1.29]
Expected change in ROE	0.24	0.44	0.63	0.85
	[6.03]	[5.84]	[5.88]	[5.98]
Accruals	0.05	0.10	0.15	0.18
	[6.42]	[6.57]	[6.54]	[6.05]
ME	0.00	0.00	0.00	0.00
	-[2.11]	-[2.47]	-[2.72]	-[3.07]
BM	0.01	0.02	0.03	0.05
	[6.15]	[6.23]	[6.45]	[6.44]
Investment	0.01	0.01	0.01	0.02
	[5.13]	[5.44]	[5.43]	[5.47]
ret(-1,0)	-0.04	-0.03	0.00	0.01
	-[5.52]	-[2.20]	[0.16]	[0.77]
ret(-12,-1)	0.01	0.02	0.01	0.00
	[2.24]	[1.60]	[0.97]	[0.17]
Adj R2	4.01%	4.18%	4.34%	4.28%
Obs.	1,365,180	1,335,352	1,306,247	1,277,939

Table VIII: Fama–MacBeth regressions: Testing the learning mechanism. (1973-2021)

Cross-sectional regressions testing whether earnings surprises (firm-level and industry-level) predict future one-month returns after controlling for profitability levels. Interaction terms with posterior uncertainty ($\nu_i \cdot dW$) show that belief-updating shocks are priced, consistent with the model predictions. Firm-level controls include: Accruals, Market value of equity, Book-market ratio, Investment, past one-month return (ret(-1,0)), and momentum (ret(-12,-1)). Newey-West adjusted t-stats are in brackets. In months where there is no belief-updating shocks due to earnings news, we set $\nu_i \cdot dW$ to zero.

	$r_{t,t+1}$	$r_{t,t+1}$	$r_{t,t+1}$	$r_{t,t+1}$	$r_{t,t+1}$	$r_{t,t+1}$
Intercept	0.00	0.00	0.00	0.00	0.00	0.00
	[1.51]	[0.43]	[0.63]	[1.67]	[0.43]	[0.55]
ROE	0.01	0.01	0.01	0.02	0.03	0.03
	[1.27]	[1.37]	[1.76]	[3.65]	[4.10]	[4.06]
Industry ROE	0.20	0.19	4.84	0.28	0.26	2.63
	[2.72]	[2.61]	[1.63]	[4.24]	[3.98]	[1.43]
$\nu_{it} * d\tilde{W}^x_{it}$	29.38	30.65	31.29	33.66	31.82	31.91
	[4.40]	[4.56]	[4.58]	[5.62]	[5.30]	[5.26]
$\nu_{it} * d\tilde{W}^s_{it}$	25.74	71.37	11.16	-28.99	-17.70	-7.67
	[1.03]	[2.88]	[0.48]	-[1.38]	-[0.84]	-[0.41]
λ_i		0.00	0.00		0.00	0.00
		[4.42]	[4.44]		[4.18]	[4.65]
κ_i		0.01	0.00		0.01	0.00
		[9.50]	[2.55]		[9.07]	[1.70]
$\hat{\mu}_{it}$			-5.30			-2.81
			-[1.56]			-[1.28]
$\bar{\mu}_i$			0.00			0.01
			-[1.23]			[3.21]
Controls	No	No	No	No	Yes	Yes
R^2_{Adj}	1.12%	1.48%	1.81%	3.64%	3.96%	4.25%
Obs.	1,907,973	1,664,057	1,602,314	1,404,742	1,380,117	1,334,035

Table IX: Spanning regressions: Expected profitability revision factor. (1973-2021)

Portfolio alphas. Results of time-series spanning regressions testing whether expected profitability changes earn abnormal returns unexplained by standard asset pricing models. This table reports the $R-W$ average return spread from two-way sorts and estimated regression coefficients, across multiple model specifications (FF5, FF6, q4, q5). Sorting firms based on expected profitability changes earn positive and significant alphas. Newey-West adjusted t-stats are in brackets.

	Expected profitability revision factor					Short-run Component					Long-run Component				
	R-W	FF5	FF6	q4	q5	R-W	FF5	FF6	q4	q5	R-W	FF5	FF6	q4	q5
alpha	0.27	0.24	0.22	0.24	0.29	0.25	0.21	0.20	0.20	0.25	0.20	0.14	0.16	0.19	0.24
	[4.15]	[4.39]	[4.05]	[4.21]	[4.76]	[4.00]	[4.03]	[3.68]	[3.62]	[4.18]	[3.28]	[2.51]	[2.84]	[3.36]	[3.87]
MKTRF		0.09	0.09	0.09	0.08		0.10	0.10	0.10	0.09		0.07	0.07	0.06	0.05
(R_MKT)		[6.95]	[7.17]	[6.74]	[5.82]		[7.78]	[8.02]	[7.75]	[6.80]		[5.53]	[5.12]	[4.79]	[4.01]
SMB		0.13	0.13	0.14	0.13		0.14	0.14	0.15	0.14		0.11	0.11	0.09	0.09
(R_ME)		[6.99]	[7.00]	[7.32]	[6.85]		[7.42]	[7.43]	[8.13]	[7.66]		[5.79]	[5.82]	[4.95]	[4.55]
HML		-0.11	-0.10	-0.13	-0.13		-0.10	-0.09	-0.12	-0.11		-0.06	-0.07	-0.01	0.00
(R_IA)		-[4.78]	-[4.04]	-[4.50]	-[4.35]		-[4.51]	-[3.74]	-[4.22]	-[4.07]		-[2.49]	-[3.03]	-[0.18]	-[0.04]
RMW		-0.07	-0.08	-0.03	0.00		-0.06	-0.06	0.00	0.02		-0.05	-0.04	-0.09	-0.07
(R_ROE)		-[2.95]	-[3.09]	-[1.28]	-[0.03]		-[2.52]	-[2.67]	-[0.20]	[0.88]		-[1.84]	-[1.68]	-[4.17]	-[2.68]
CMA		0.01	0.00	0.00	-0.08		0.01	0.00		-0.07		0.09	0.10	0.00	-0.07
(R_EG)		[0.32]	[0.06]	[0.00]	-[2.24]		[0.17]	-[0.10]		-[2.17]		[2.44]	[2.72]	[0.00]	-[2.00]
MOM			0.02					0.02				0.00	-0.03		
			[1.85]					[1.99]				[0.00]	-[2.19]		
R^2_{Adj}		0.30	0.31	0.27	0.27		0.32	0.33	0.29	0.30		0.16	0.17	0.16	0.17

A. Notation and Estimation summary

Table A.1 summarizes the main notation used in the paper.

TABLE A.1: **Notation summary.** Key variables and parameters used throughout the paper.

Symbol	**Definition / interpretation**
X_{it}	Realized firm i profitability (e.g., ROE) observable at quarter t
μ_{it}	Latent profitability driver (fundamental)
s_{it}	Observable industry-level profitability signal available at t
$\mathcal{F}_{it}$	Information set available for firm i at quarter t
$\hat{\mu}_{it}$	Posterior belief (filtered mean) about the latent driver μ_{it}
ν_{it}	Posterior uncertainty (filtered variance) about μ_{it}
λ_i	Mean-reversion speed of realized profitability X_{it}
κ_i	Mean-reversion speed of the latent driver μ_{it}
θ_{it}	Vector of learning parameters estimated for firm i using data up to t
$\mathcal{W}_{it}$	Expanding estimation window ending at t
ExpProf_{it}	Model-implied expected profitability measure dated t
$\Delta\text{ExpProf}_{it}$	Expected profitability revision (expected change relative to current level)

For transparency and replicability, we provide a step-by-step description of how we construct expected profitability, expected profitability revisions, and the inputs used in the empirical tests. Throughout, all quantities dated t are constructed using only information observable by quarter t.

1. **Sample and timing conventions.** We work at the quarterly frequency. For each firm i and quarter t, we define the accounting observation date as the quarter in which the relevant quarterly accounting statement is first observable. We require the firm to have valid quarterly income and book equity data to compute ROE, and we require an industry classification (Fama-French 30).

2. **Compute realized profitability X_{it} (firm ROE).** For each firm-quarter, we compute realized profitability as
$$X_{it} \equiv \frac{\text{IBQ}_{it}}{\text{BEQ}_{i,t-1}},$$
where IBQ_{it} is quarterly income before extraordinary items and $\text{BEQ}_{i,t-1}$ is lagged book equity. We winsorize X_{it} at the 1% and 99% levels each quarter and drop observations

with nonpositive or missing denominators. We treat the resulting X_{it} as observable at quarter t.

3. **Compute the industry profitability signal s_{it} in real time.** Let $\mathcal{I}(i,t)$ denote firm i's industry at quarter t and let $\mathcal{S}_{\mathcal{I}(i,t),t}$ denote the set of firms in that industry whose ROE is observable by time t. We construct an industry benchmark in real time by aggregating only information that has been reported and is available as of t. Specifically, for each industry we retain only positive ROE observations and compute the cross-sectional median:

$$s_{it} \equiv \text{median}\left\{X_{jt} : j \in \mathcal{S}^{+}_{\mathcal{I}(i,t),t}\right\}, \qquad \mathcal{S}^{+}_{\mathcal{I}(i,t),t} \equiv \left\{j \in \mathcal{S}_{\mathcal{I}(i,t),t} : X_{jt} > 0\right\}.$$

   We update this signal at a monthly frequency. In month t, we anchor the computation to the most recently completed fiscal quarter and include all firms in the industry that have reported ROE for that quarter by month-end. Operationally, for each month-end t_k we collect all firm-quarter ROE observations with fiscal quarter-end equal to the previous quarter-end and reporting date (RDQ) no later than t_k. This yields an up-to-date, real-time measure of the industry benchmark ROE that incorporates newly released earnings within the quarter as they arrive. We verify robustness to alternative industry definitions (e.g., FF12, FF30, SIC-based) and to alternative aggregation (mean versus median).

4. **Define the expanding estimation window and re-estimation schedule.** For each firm i and estimation quarter t, we define an expanding sample window

$$\mathcal{W}_{it} \equiv \{t_0, \ldots, t\},$$

   where t_0 is the first quarter in which firm i enters the sample (or the first quarter with sufficient data for estimation). Thus, the window length grows over time as new observations become available. We estimate parameters at a coarse frequency (every two years, i.e., every eight quarters). Let $\tau(t) \leq t$ denote the most recent estimation quarter for firm i; between re-estimation dates, we carry forward the most recent estimate $\theta_{i,\tau(t)}$.

5. **Estimate learning parameters θ_{it} (SMM).** At each estimation quarter t, we estimate the parameter vector

$$\theta_{it} \equiv (\lambda_{it}, \kappa_{it}, \sigma_{\mu,it})$$

using only data in $\mathcal{W}_{it}$. Within $\mathcal{W}_{it}$, we compute the three target moments used in the paper: (i) the autocorrelation of firm ROE, (ii) the autocorrelation of industry ROE, and (iii) the covariance between firm and industry ROE. We simulate the model under candidate θ and choose θ_{it} to minimize the SMM objective:

$$\theta_{it} \in \arg\min_{\theta\in\Theta} \left(m_{it}^{\text{sim}}(\theta) - m_{it}^{\text{data}}\right)' W_{it} \left(m_{it}^{\text{sim}}(\theta) - m_{it}^{\text{data}}\right),$$

where m_{it}^{data} is computed from observed $(X_{i\tau}, s_{i\tau})$ for $\tau \in \mathcal{W}_{it}$ and $m_{it}^{\text{sim}}(\theta)$ is computed from simulated paths using the same window length. Auxiliary quantities used for initialization are computed within $\mathcal{W}_{it}$: we set $\bar{\mu}_{it}$ to the window mean of $X_{i\tau}$, $\sigma_{X,it}$ to the window standard deviation of $X_{i\tau}$, and $\sigma_{s,it}$ to the window standard deviation of $s_{i\tau}$, all for $\tau \in \mathcal{W}_{it}$.

6. **Run the recursive filter to obtain beliefs $\hat{\mu}_{it}$ and innovations.** Starting from an initial belief $\hat{\mu}_{i,t_0}$ at the first quarter t_0 with valid (X_{it}, s_{it}), we apply the model-implied updating equations recursively. For each quarter $t \geq t_0$, we use only current observables (X_{it}, s_{it}) and the most recent parameter estimate $\theta_{i,\tau(t)}$ (with $\tau(t) \leq t$) to update beliefs. This recursion yields: (i) the posterior mean belief $\hat{\mu}_{it}$ and (ii) the model-implied innovations (earnings surprises) associated with the firm and industry signals. All outputs at quarter t are measurable with respect to $\mathcal{F}_{it}$ by construction.

7. **Construct expected profitability and expected profitability revisions.** Given $\hat{\mu}_{it}$ and $\theta_{i,\tau(t)}$, we compute the model-implied expected profitability object ExpProf_{it} and the expected profitability revision (expected change relative to current profitability),

$$\Delta\text{ExpProf}_{it} \equiv \mathbb{E}_t[X_{i,t+1}] - X_{it}$$

When reported, we further decompose ExpProf_{it} and $\Delta\text{ExpProf}_{it}$ into a short-run updating component and a long-run anchoring component using the model's closed-form mapping.

8. **Align signals with returns for monthly portfolio formation and regressions.** All empirical tests are conducted at a monthly frequency and follow a strict real-time convention. At each month-end t, we form signals using only information available as of t and align them with returns realized strictly after t.

The industry profitability signal is updated monthly: at each month-end, we compute the most up-to-date industry benchmark using only ROE observations that have been reported by that date (see the signal construction above). Firm-level ROE is taken to be the most recently observed value available at month-end t (i.e., we carry forward the latest reported firm ROE until a new report arrives).

In the baseline portfolio tests, we sort stocks into quantiles using $\Delta\text{ExpProf}_{it}$ (and/or its components) computed from information available at month-end t and hold the portfolio over the subsequent month (or the monthly horizon specified in the paper). In predictive regressions, we regress next-month (or h-month-ahead) returns on signals dated t, including standard controls and fixed effects as appropriate.

B. Appendix: Filtering Theorem

We follow Theorem 12.2 from Liptser and Shiryaev (2013) and define a system represented by a set of observables s_t and unobservable θ_t with the following dynamics:

$$d\theta_t = (a_0(t) + a_1(t)\theta_t + a_2(t)s_t)\,dt + \sum_{i=1}^{2} b_i(t)dW_i(t) \tag{B.1}$$

$$ds_t = (A_0(t) + A_1(t)\theta_t + A_2(t)s_t)\,dt + \sum_{i=1}^{2} B_i(t)dW_i(t), \tag{B.2}$$

the posterior beliefs about θ_t, defined as $\hat{\theta}_t$ and the posterior uncertainty ν_t evolve according to

$$d\hat{\theta}_t = \left(a_0(t) + a_1(t)\hat{\theta}_t + a_2(t)s_t\right)dt + \left[(b \circ B) + \nu_{it}A_1'(t)\right](B \circ B)^{-1}\left[ds_t - \left(A_0(t) + A_1(t)\hat{\theta}_t + A_2(t)s_t\right)dt\right], \tag{B.3}$$

and

$$d\nu_t = \left[a_1(t)\nu_t + \nu_t a_1'(t) + (b \circ b) - ((b \circ B) + \nu_t A_1'(t))(B \circ B)^{-1}((b \circ B) + \nu_t A_1'(t))\right]dt. \tag{B.4}$$

Our set of observables, $s_t = (X_{it}, s_{it})'$, is described by the following dynamics

$$\begin{bmatrix} dX_{it} \\ ds_{it} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{A_0} + \underbrace{\begin{bmatrix} \lambda_i \\ 1 \end{bmatrix}}_{A_1} \mu_t + \underbrace{\begin{bmatrix} -\lambda_i & 0 \\ 0 & 0 \end{bmatrix}}_{A_2} \begin{bmatrix} X_{it} & s_{it} \end{bmatrix} \right) dt + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{B_1} dW_t^{\mu} + \underbrace{\begin{bmatrix} \sigma_{ix} & 0 \\ 0 & \sigma_{is} \end{bmatrix}}_{B_2} \begin{bmatrix} dW_{it}^X \\ dW_{it}^s \end{bmatrix}, \tag{B.5}$$

and the unobservable in our model is the firm profitability μ_{it}, i.e., $\theta_t = \mu_{it}$, which evolves according to

$$d\mu_{it} = \left(\underbrace{\kappa_i}_{a_0} \bar{\mu}_i \underbrace{-\kappa_i}_{a_1} \mu_{it} \right) dt + \underbrace{\sigma_\mu}_{b_1} dW_{it}^{\mu} + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{b_2} \begin{bmatrix} dW_{it}^X \\ dW_{it}^s \end{bmatrix}. \tag{B.6}$$

Applying Theorem 12.2 yields the following dynamics for the posterior beliefs $\hat{\mu}_{it}$, $\hat{\mu}_{it} = E_t(\mu_{it}|\mathcal{F}_t)$

$$d\hat{\mu}_{it} = \kappa_i(\bar{\mu}_i - \hat{\mu}_{it})dt + \frac{\lambda_i \nu_{it}}{\sigma_{ix}} d\tilde{W}_{it}^x + \frac{\nu_{it}}{\sigma_{is}} d\tilde{W}_{it}^s, \tag{B.7}$$

where

$$d\tilde{W}_{it}^x = \frac{dX_{it} - \lambda_i(\hat{\mu}_{it} - X_{it})dt}{\sigma_{ix}}, \tag{B.8}$$

and

$$d\tilde{W}_{it}^s = \frac{ds_{it} - \hat{\mu}_{it}dt}{\sigma_{is}}. \tag{B.9}$$

Posterior uncertainty ν_{it}, $\nu_{it} = E_t([\mu_{it} - \hat{\mu}_{it}]^2)$ evolves according to

$$d\nu_{it} = \left(\sigma_{\mu_i}^2 - 2\kappa_i \nu_{it} - \frac{\lambda_i^2 \nu_{it}^2}{\sigma_{ix}^2} - \frac{\nu_{it}^2}{\sigma_{is}^2} \right) dt. \tag{B.10}$$

C. Proofs

Proof of Theorem 3.2

Proof. Posterior beliefs $\hat{\mu}_{it}$ and posterior uncertainty ν_{it} evolve according to

$$d\hat{\mu}_{it} = \kappa_i(\bar{\mu}_i - \hat{\mu}_{it})dt + \frac{\lambda_i \nu_{it}}{\sigma_{ix}} d\tilde{W}_{it}^x + \frac{\nu_{it}}{\sigma_{is}} d\tilde{W}_{it}^s, \tag{C.1}$$

and

$$d\nu_{it} = \left(\sigma_{\mu_i}^2 - 2\kappa_i \nu_{it} - \frac{\lambda_i^2 \nu_{it}^2}{\sigma_{ix}^2} - \frac{\nu_{it}^2}{\sigma_{is}^2}\right) dt. \tag{C.2}$$

To derive the conditional expectation of future firm-level profitability at time τ, $X_{i\tau}$: $E_t(X_{i\tau})$, we first define the conditional expectation of future μ_{is}, formed at t, $\forall s > t$:

$$E_t(\mu_{is}) = \bar{\mu}_i + (\hat{\mu}_{it} - \bar{\mu}_i)e^{-\kappa_i(s-t)}. \tag{C.3}$$

Next, we define $y_t = e^{\lambda_i t} X_{it}$, with the following properties.

$$dy_t = e^{\lambda_i t}(dX_{it} + \lambda x_{it} dt) = e^{\lambda_i t}(\lambda_i \mu_{it} dt + \sigma_{ix} dW_{it}^x)$$

$$y_\tau = y_t + \lambda_i \int_t^\tau e^{\lambda_i s} \mu_{is} ds + \sigma_{ix} \int_t^\tau e^{\lambda_s s} dW_{is}^x$$

The expected value of y_τ is given by

$$E_t(y_\tau) = y_t + \lambda_i \int_t^\tau e^{\lambda_i s} E_t(\mu_{is}) ds = y_t + \lambda_i \int_t^\tau e^{\lambda_i s}\left(\bar{\mu}_i + (\hat{\mu}_{it} - \bar{\mu}_i)e^{-\kappa_i(s-t)}\right) ds$$

$$E_t(y_\tau) = y_t + \lambda_i \bar{\mu}_i \frac{1}{\lambda_i}\left[e^{\lambda_i \tau} - e^{\lambda_i t}\right] + \frac{\lambda_i(\hat{\mu}_{it} - \bar{\mu}_i)}{\lambda_i - \kappa_i}\left(e^{(\lambda_i - \kappa_i)\tau + \kappa_i t} - e^{\lambda_i t}\right),$$

which translates into

$$E_t(X_{i\tau}) = \underbrace{X_{it} e^{-\lambda_i(\tau - t)}}_{\lim_{t\to-\infty}=0} + \bar{\mu}_i \left[1 - \underbrace{e^{-\lambda_i(\tau-t)}}_{\lim_{t\to-\infty}=0}\right] + \frac{\lambda_i(\hat{\mu}_{it} - \bar{\mu}_i)}{\lambda_i - \kappa_i} \underbrace{\left[e^{-\kappa_i(\tau-t)} - e^{-\lambda_i(\tau-t)}\right]}_{\lim_{t\to-\infty}=0}. \tag{C.4}$$

This expectation converges to the long-run mean $\bar{\mu}_i$. □